Smart Structures and Systems, *Vol. 22, No. 2 (2018) 193-200*
DOI: https://doi.org/10.12989/sss.2018.22.2.193

Reliability analysis for fatigue damage of railway welded bogies using Bayesian update based inspection

Fang-Jun Zuo[1,2], Yan-Feng Li[*3] and Hong-Zhong Huang[3]

[1]*School of Mechanical and Electrical Engineering, University of Electronic Science and Technology of China*
No. 2006, Xiyuan Avenue, West Hi-Tech Zone, Chengdu, Sichuan, 611731, P.R. China
[2]*College of Mechanical engineering, Chengdu Technological University*
No. 1, Zhongxin Avenue, Pidu Distric, Chengdu, Sichuan, 611730, P.R. China
[3]*Center for System Reliability and Safety, University of Electronic Science and Technology of China*
No. 2006, Xiyuan Avenue, West Hi-Tech Zone, Chengdu, Sichuan, 611731, P.R. China

(Received May 8, 2017, Revised March 15, 2018, Accepted March 19, 2018)

Abstract. From the viewpoint of engineering applications, the prediction of the failure of bogies plays an important role in preventing the occurrence of fatigue. Fatigue is a complex phenomenon affected by many uncertainties (such as load, environment, geometrical and material properties, and so on). The key to predict fatigue damage accurately is how to quantify these uncertainties. A Bayesian model is used to account for the uncertainty of various sources when predicting fatigue damage of structural components. In spite of improvements in the design of fatigue-sensitive structures, periodic non-destructive inspections are required for components. With the help of modern nondestructive inspection techniques, the fatigue flaws can be detected for bogie structures, and fatigue reliability can be updated by using Bayesian theorem with inspection data. A practical fatigue analysis of welded bogies is utilized to testify the effectiveness of the proposed methods.

Keywords: bogies fatigue; Bayesian; uncertain; crack growth; nondestructive inspection

1. Introduction

As one of the critical safety components of rail vehicles, the bogie frame is the main load-bearing and power transmission components. When the vehicle is in service, the bogies not only need to withstand loads, but also need to pass a variety of forces between the body and the wheel. Materials aging, through the evolution and accumulation of fatigue damage, is one of the dominate factors to decrease the reliability and safety of bogies, whose failure often lead to derailments, deaths and injuries. Therefore, fatigue life prediction and reliability evaluation are critical for the design and maintenance of the bogie frame. For the analysis and design of critical components, fatigue life prediction and reliability assessment are still challenging tasks despite extensive research during the past several decades (Mi *et al.* 2018, Li *et al.* 2018). There is a considerable interest in developing an approach to predict the lifetime through probabilistic modeling of fatigue, particularly for the critical components, such as bogie frame and railway axle, which service in harsh environments (Dong 2001, Liu and Mahadevan 2005, Richard and Andrew 2007, Li and Guo 2015, Huang *et al*. 2017).

Existing models focus on the deterministic fatigue crack growth process. However, the fatigue process of bogie frame/components in service is stochastic in nature. The fatigue crack growth is a stochastic process affected by uncertainties from many sources. Previous researches revealed that the uncertainties can be divided into the following groups (Zhang 2000, Sankararaman *et al.* 2010, Mahadevan and Rebba 2006): material properties, structural properties, load variation, parameter estimation, and model error. The first three categories represent inherent variability through random variables, whereas the last two categories focus on the uncertainties associated with models and parameters selection. How to quantify these uncertainties is the key to accurately predict fatigue life. Many researchers have worked on uncertainty quantification of some aspects of the damage tolerance problems. A systematic analysis should incorporate multiple sources and multiple types of uncertainty that are inherent in the fatigue crack growth modeling procedure; however, such an analysis has not been completed. In this paper, major work is to quantify the model and the parameters related to the fatigue crack growth problem. In addition, stochastic variable amplitude loading conditions will be considered and addressed in an effort to more realistically represent in-service loading conditions.

Practical engineering materials and structures have defects and cracks in nature (Huang and Yang 2008, Ghodrati *et al.* 2011, Xiang *et al*. 2012, Zheng *et al*. 2018). Cyclic loading can cause defects to nucleate cracks and existing cracks to propagate through fatigue processes. A problem in the fracture mechanics-based life prediction is to determine the initial crack size in the crack growth analysis (Sankararaman *et al.* 2010). A method to solve this problem

∗Corresponding author, Associate Professor
E-mail: hzhuang@uestc.edu.cn

Copyright © 2018 Techno-Press, Ltd.
http://www.techno-press.com/journals/sss&subpage=7
ISSN: 1738-1584 (Print), 1738-1991 (Online)

is assuming the crack length empirically (Makeev *et al.* 2007, Cross *et al.* 2007), and an alternative method is to use the results from non-destructive inspection (NDI). To date, many non-destructive inspection methods, including X-ray, ultrasonic waves, and electric potential drop methods, can be employed to detect cracks. Once cracks are detected, fracture mechanics methods can be used to determine the extended life of the structures or components (Luo and Bowen 2003, Sankararaman *et al.* 2011). Moreover, the data obtained from the inspections also can be combined with these models to update the reliability during the remaining service life by the Bayesian approach.

Fracture mechanics-based models for crack growth analysis have been proposed to predict the performance of the component. These models, such as Paris law, Walker model, and Forman model, etc. were calibrated from experimental tests of coupons. Each model has its own advantage in applications. However, there is not a universal criterion to decide which model is more suitable. The Bayesian theory is presented to quantify uncertainty of models and provide a more reasonable choice in decision-making, which can contribute to minimize the prediction errors originating from the unreasonable models.

The Bayesian approach can potentially provide more accurate estimations by combining evidences, such as test data with prior knowledge available from theoretical analyses and/or previous experimental results, which can reduce required tests and save time and resources (Gelman *et al.* 2004, Li *et al.* 2015). The Bayesian inference is a high-efficiency technique that can update a given state of knowledge. Some researchers have addressed uncertainty in fatigue modeling by Bayesian methods. Mahadevan and Rebba (2006) developed the idea of updating failure probability based on the information of non-destructive inspection (NDI) with the Bayesian approach. Zhao *et al.* (1994) investigated the effect of the uncertainties of detection on updating and used the updated reliability index in inspection schedule, maintenances and repair decisions. Cross *et al.* (2007) used Bayesian inference to estimate parameters underlying crack growth behavior. Sankararaman *et al.* (2009, 2011) established dynamic Bayesian networks for model parameter estimation and calculated Bayes factors to quantify model uncertainty. Zheng and Ellingwood (1998) incorporated a time-dependent noise term to the fatigue crack growth model to deal with a wide-band load process and considered the interaction of corrosion and fatigue/fracture damage, and used the outcome of NDI to update the distribution of crack size. Fatemi and Yang (1998) gave a survey of the state of the art for cumulative fatigue damage and life prediction theories of homogenous materials. Chiachío *et al.* (2015) proposed a stochastic model for damage evolution and used a Bayesian model selection framework to account for model uncertainty.

The rest of the paper is organized as follows: Section 2 discusses the crack growth models and sources of uncertainty. In Section 3, the Bayesian inference is explained in the view of updating distribution of fatigue life based on the inspection data. In Section 4, the proposed methods are demonstrated for fatigue analysis of bogies. Finally, the conclusions are drawn in Section 5.

2. Fatigue crack growth modeling and reliability analysis

Fatigue cracks generally appear on the surface of material or at large inclusions, resulting from high stresses, surface roughness, fretting, corrosion, etc. Fatigue crack growth on a macroscopic level usually occurs perpendicular to the main or principal stress, which is dependent on the material parameters, the material thickness, and the orientation of the crack relative to principal material directions. Furthermore, the crack growth is dominated by the cyclic stress amplitude, the mean stress and the environment.

Linear elastic fracture mechanics assumes that all structures contain flaws. Cracks grow from the initial size a_0 to the critical size a_c. Fatigue crack propagation occurs as a result of cyclic loading conditions with cracks growing a given increment (Δa) in a given number of loading cycles (ΔN). When the crack size reaches a critical level, the crack growth becomes unstable and failure occurs. According to linear elastic fracture mechanics, the plastic deformation near the crack tip is controlled by the stress intensity factor range, and can provide the small scale yielding condition applicably. Various deterministic fatigue crack growth rate functions have been proposed in the literature. The functions can be represented by a general form (Sankararaman *et al.* 2010, Mahadevan and Rebba 2006, Zhou *et al.* 2017, Huang *et al.* 2014)

$$\frac{da(t)}{dt} = f(\Delta K, K_{\max}, K_{\mathrm{th}}, R, S, a, \cdots) \tag{1}$$

where a is the crack length, $a(t)$ is the crack length at time t, ΔK is the stress intensity factor range, $K_{\max}$ is the spectrum peak stress intensity factor, S is the fatigue strength, or stress amplitude, or peak stress level in the loading spectrum, $da(t)/dt$ is crack growth per cycle and $f(\Delta K, K_{\max}, R, S, a, \cdots)$ is a non-negative function.

Throughout decades of investigation, numerous fatigue models have been proposed, such as Paris model, Walker model, Forman model, and generalized Forman model, are commonly used (Weertman 1966, Zhang and Mahadevan 2000, Sankararaman *et al.* 2010).

Paris model

$$\frac{da}{dN} = C(\Delta K)^n \tag{2}$$

Walker model

$$\frac{da}{dN} = C(\Delta K)^n (1-R)^m \tag{3}$$

Forman model

$$\frac{da}{dN}=C\frac{(\Delta K)^n}{(1-R)K_C-\Delta K} \tag{4}$$

Generalized Forman model

$$\frac{da}{dN}=C\left[\left(\frac{1-f_0}{1-R}\right)\Delta K\right]^{m_1}\frac{\left(1-\dfrac{\Delta K_{th}}{\Delta K}\right)^{m_2}}{\left(1-\dfrac{\Delta K}{(1-R)K_C}\right)^{m_3}} \tag{5}$$

where C, n, m are the material constants, R is the stress ratio, ΔK_{th} is the crack threshold, K_c is the plane stress fracture toughness of material dependent on the thickness of structures. f_0 is the fatigue crack opening function, which can be determined as

$$f_0=\frac{K_{\text{open}}}{K_{\max}}=\begin{cases}\max(R, A_0+A_1R+A_2R^2+A_3R^3) & R\geq 0\\ A_0+A_1R & -2\leq R<0\end{cases} \tag{6}$$

where

$$A_0=(0.825-0.24\alpha_0+0.05\alpha_0{}^2)\left[\cos\left(\frac{\pi S}{2\delta_0}\right)\right]^{\frac{1}{\alpha_0}}$$

$$A_1=(0.415-0.71\alpha_0)\frac{S}{\delta_0}$$

$$A_2=(1-A_0-A_1-A_3)$$

$$A_3=2A_0+A_1-1$$

where α_0 is the plane stress/strain constraint factor, and S/δ_0 is the ratio of maximum stress to the flow stress.

According to elastic fracture mechanics, the stress intensity factor K is the product of both functions concerning stress S and crack size a

$$K=X(S)Y(a) \tag{7}$$

where $Y(a)=\alpha(a)\sqrt{\pi a}$, $\alpha(a)$ is the geometry correction function of fatigue crack. From Eq. (7), the mean and range of stress intensity factor are

$$K_{\text{m}}=X(S_{\text{m}})Y(a) \tag{8}$$

$$\begin{aligned}\Delta K=K_{\max}-K_{\min}&=\left[X(S_{\max})-X(S_{\min})\right]\cdot Y(a)\\&=\Delta X\cdot Y(a)\end{aligned} \tag{9}$$

In the case of considering the correction of plastic zone near crack tip, $X(S)$ can be obtained as

$$X(S)=\frac{S}{\sqrt{1-\pi\kappa(S/\delta_s)}}$$

$$\kappa=\begin{cases}1/2\pi, & \text{under plane stress state}\\ (1-2v)^2/2\pi, & \text{under plane strain state}\end{cases} \tag{10}$$

where δ_s is the yield limit, κ is the material constant, and v is the Poisson ratio. Since a fatigue stress cycle is defined by two stress components of amplitude S_{a} and mean S_m, and $S_{\max}=S_{\text{a}}+S_{\text{m}}$, $S_{\min}=S_{\text{m}}-S_{\text{a}}$, then

$$\begin{aligned}\Delta X&=X(S_{\max})\text{-}X(S_{\min})\\&=\frac{S_{\text{a}}+S_{\text{m}}}{\sqrt{1-\pi\alpha\left[(S_{\text{a}}+S_{\text{m}})/\delta_{\text{s}}\right]^2}}-\frac{S_{\text{m}}-S_{\text{a}}}{\sqrt{1-\pi\alpha\left[(S_{\text{m}}-S_{\text{a}})/\delta_{\text{s}}\right]^2}}\end{aligned} \tag{11}$$

Substituting Eq. (11) into Eq. (9), Eq. (9) can be rewritten as

$$\Delta K=\left\{\frac{S_{\text{a}}+S_{\text{m}}}{\sqrt{1-\pi\alpha\left[(S_{\text{a}}+S_{\text{m}})/\delta_{\text{s}}\right]^2}}-\frac{S_{\text{m}}-S_{\text{a}}}{\sqrt{1-\pi\alpha\left[(S_{\text{m}}-S_{\text{a}})/\delta_{\text{s}}\right]^2}}\right\}Y(a) \tag{12}$$

From the above descriptions, several empirical fatigue models are investigated to describe typical crack growth behavior in metals in the present literature. Each model has its own advantage in applications. The Paris model is most commonly used in fatigue analysis for its simplicity. But it only considers the intermediate region of crack growth and assumes that the crack growth rate depends on the stress-intensity range only (Paris 1964). According to Soares and Garbatov (1999), for a particular problem, in principle only one model is the suitable one. However, there is not a universal criterion to decide which model is more suitable. The Bayesian theory is presented to quantify uncertainty of models. Edwards (1984), Soares and Garbatov (1999) employed the Bayesian approach to describe a random variable by taking several competing probability distribution types into consideration. As to the reliability estimation problem, this paper shows that the Bayesian framework can not only consider multiple competing distribution types and multiple possible sets of parameters within each distribution, but can also simultaneously consider multiple competing limit state formulations or it possibly appropriate for the same problem.

Integrating Eq. (1) with respect to crack size from a_1 to a_2 corresponding to the number of stress cycles N_1 and N_2, we have

$$\begin{aligned}&\int_{a_1}^{a_2}\frac{1}{f(\Delta K, K_{\max}, R, S, a)}da\\&=\int_{N_1}^{N_2}CS^m dN=C\bar{S}^m(N_2-N_1),\end{aligned} \tag{13}$$

where $\bar{S}^m$ is the mean stress range, and it can be evaluated as

$$\bar{S}^m=\int_0^a S^m f_s(S)dS \tag{14}$$

where $f_s(S)$ is the probability density function (PDF) of the stress range parameter S. Assuming that the stress range follows a Rayleigh distribution, which is used to describe the parts, and the components are subjected unstable cyclic stress. The mean stress effect can be represented as follows

$$\bar{S}^m=\left(\sqrt{2}S_0\right)^m\Gamma^m\left(\frac{m}{2}+1\right) \tag{15}$$

where $\Gamma(\ )$ is the Gamma function and S_0 is a statistical parameter. Based on Eqs. (13) and (15), the fatigue crack propagation life can be determined.

The fatigue damage accumulation function represented by Eq. (1) can be alternatively expressed as (Zhang and Mahadevan 2000)

$$\psi(a_2,a_1)=\int_{a_1}^{a_2}\frac{1}{f(\Delta K,K_{\max},R,S,a)}da \qquad (16)$$

Since Eq. (16) is a monotonically increasing function of the crack size, the limit state function represented by Eq. (17) can be rewritten as

$$g(z)=\psi(a_c,a_0)-C\bar{S}^m(N_c-N_0) \qquad (17)$$

Eq. (17) states that the initial size a_0 of crack has propagated to a critical size a_c from N_0th to N_cth stress cycle.

The corresponding failure probability can be calculated as

$$p_f=p(g(z)\le 0)\cong\Phi(-\beta) \qquad (18)$$

where β is the reliability index. The goal of a reliability-based fatigue analysis procedure is to keep the reliability index above a reassigned value during the service life of the bogies.

3. Bayesian description and updating of model

As mentioned above, numerous crack growth rate empirical models have been proposed within the literature (e.g., Paris, Walk, Foreman, etc.), which contains different parameters obtained by experimental tests, and few models can be applied universally to all fatigue crack growth problems. Every model has its own limitations and uncertainties; and cannot demonstrate which one is better than others (Zhang 2000, Sankararaman *et al.* 2010, Mahadevan and Rebba 2006, Cross and Makeev 2007). A Bayesian model is proposed to account for uncertainty of various sources. Bayesian inference estimates the degree of belief in a hypothesis based on the collected evidence. Bayes (1763) formulated the degree of belief according to the identity in conditional probability. Some researchers have addressed uncertainty in fatigue modeling in accordance with Bayesian methods, mostly focused on crack propagation in metals. Soares and Garbatov (1999) demonstrated that the use of all available information through the Bayesian prediction shows a much smaller variability than that of individual model. Depending on how the various models are weighted, different final values will be obtained but their range of variation is much smaller than that in the case of individual predictions. Edwards (1984) also used the Bayesian approach to research a structural system subjected to dynamic loadings that could be described by a normal, a lognormal or a Weibull distribution. Note that the Bayesian approach is an effective way to deal with small sample problems. Due to the data insufficiency and uncertainty of prediction models, it is unreasonable to predict the fatigue damage with a deterministic model.

Thus, in situations where physical, model and statistical uncertainties are equal importance, an intuitively appealing and more logical approach would be to use a “weighted average” of all possible models, and they sets within each model (Rebba 2005). The method to describe and update these uncertainties is formulated as follows.

Consider a fatigue crack growth model

$$N=\int_{a_1}^{a_c}\frac{1}{F(\Delta K,K_{\max},R,S,a)}da=f(a,\theta) \qquad (19)$$

where the input variable a_c is the critical crack size, the output variable N is fatigue life to the final crack size, θ is the vector of model parameters.

Suppose there are a set of models of the fatigue crack propagation $M1$, $M2$,…M_k. If each model is possible candidates, the Bayesian probability of event D, which incorporates both parameter and model uncertainty denoted by

$$P(D)=\sum_{i=1}^{K}p(M_i)\int_{\theta_i}p(D|\theta_i,M_i)f\left(\theta_i|M_i\right)d\theta_i \qquad (20)$$

where $p(M_i)$ refers to the model uncertainty, and is the prior probability assigned to model i . $\theta_i=\{\theta_{i1},\theta_{i2},\theta_{i3},\cdots,\theta_{in}\}^{\mathrm{T}}$ refers to the vector of distribution parameters within the model M_i, which includes the initial flaw size, geometry, loading, residual stress, and material properties.

$P(H_i)$ is the prior probability of model

$$P(H_i)=P(M_i)\times p(\theta_i|M_i) \qquad (21)$$

After getting an inspection of fatigue crack length d_a, the posterior probabilities are then given by Bayes theorem, we have

$$P(H_i|d_a)=P(M_i|d_a)\times f\left(\theta_i|M_i,d_a\right)=\frac{p(d_a|H_i)\cdot f\left(\theta_i|M_i\right)P(M_i)}{\sum_{i=1}^{m}P(M_i)\times\int_\theta p(x|\theta_i,M_i)\cdot f\left(\theta_i|M_i\right)d\theta_i} \qquad (22)$$

The posterior probability of model M_i is obtained by integrating $f(\theta_i|M_i)$

$$P(M_i|d_a)=\frac{P(M_i)\times\int_\theta p(d_a|H_i)\cdot f\left(\theta_i|M_i\right)d\theta_i}{\sum_{i=1}^{m}P(M_i)\times\int_\theta p(d_a|\theta_i,M_i)\cdot f\left(\theta_i|M_i\right)d\theta_i} \qquad (23)$$

Then the posterior distribution of θ_i in model $|M_i$ is given by Eq. (24)

$$f\left(\theta_i|M_i,d_a\right)=\frac{p(d_a|H_i)\cdot f\left(\theta_i|M_i\right)}{\int_\theta p(d_a|H_i)\cdot f(\theta_i|M_i)d\theta_i} \qquad (24)$$

It can be seen from Eq. (24) that the parameter updating in model M_i is independent of $p(M_i)$, and is updated in the same way when only the model M_i is assigned to the problem.

Furthermore, Bayesian expectation of any event E related to H_i can be derived based on the posterior model probability and parameter distribution.

$$P(E)=\sum_{i=1}^{M}P(M_i|d_a)\int_{\theta}p_i(E|H_i)f(\theta_i|M_i,d_a)d\theta_i \tag{25}$$

According to the above analysis, the Bayesian method can combine multiple mechanical models and statistical models, and it gives a more reasonable, comprehensive method for reliability prediction.

During the lifetime of bogies, periodic nondestructive inspections (NDI) are essential and important for fatigue damage evaluation, scheduling maintenance and repair. Every NDI method has its own characteristics. For a particular NDI technique, several factors are expected to affect inspection results, including modeling effects, human factors and inspection factors. All these factors add uncertainties to the inspection outcomes, and need to be considered explicitly in every mathematical model. The additional uncertainty needs to be incorporated in every mathematical model. Defining a_d as the crack detectability for a particular NDI, the results of an inspection belongs to one of the following cases: (1) no crack detection; (2) crack detection without measurement; and (3) crack detection with measured crack size A (Zhang and Mahadevan 2000, Mahadevan and Rebba 2006). The Bayesian updating method is used in this study. This is briefly discussed below in the three inspection events.

Case I : No crack detection

This implies that the actual crack size a_i at the time of inspection (corresponding to N_i stress cycles) is smaller than the minimum detectable crack size a_d for a given inspection technique. The event D can be expressed as

$$I=\psi(a_d,a_0)-C\bar{S}^m(N_d-N_0)\le 0 \tag{26}$$

Using the Bayesian approach, the model weight and probability distribution of the jth random variable $x_{j,i}$ associated with the model i are updated as

$$P(M_i|D)=\frac{P(M_i)P(I_i\le 0)}{\sum_{i=1}^{k}P(M_i)P(I_i\le 0)} \tag{27}$$

$$F_{X_{j,i}}(x_{j,i}|P(g(z)\le 0))=\frac{p(X_{j,i}-x_{j,i}\le 0\bigcap P(I_i))}{P(I_i\le 0)} \tag{28}$$

According to the posterior model weights and parameter probability distribution, the failure probability can be updated by Eq. (28). However, the failure probability can be updated alternatively in accordance with the posterior model weight $P(M_i|D)$ as

$$P_{f,up}=\sum_{i=1}^{k}P(M_k|D)\frac{P(g_k\le 0\bigcap I_k\le 0)}{P(I_k\le 0)} \tag{29}$$

Case II: Crack detected but size not measured

The event of crack detection without size measurement can be similarly represented as

$$D=-I=C\bar{S}^m(N_d-N_0)-\psi(a_d,a_0)\le 0 \tag{30}$$

Since this inspection event is complementary to that of no crack detection, Eq. (28) can still be used to update the probability of failure.

Case III: Crack detection with crack size a_d

Assuming that a crack of size a_d is measured during an inspection, the event D_A can be expressed as

$$D_A=C\bar{S}^m(N_d-N_0)-\psi(a_d,a_0)=0 \tag{31}$$

The updated probability of failure can be shown to be

$$P_{f,up}=\sum_{i=1}^{k}P(M_i|D_d)\frac{\frac{\partial P_i}{\partial a_d}(g_1\le 0\bigcap C\bar{S}^m(N_d-N_0)-\psi(a_d,a_0)\le 0)}{\frac{\partial P_i}{\partial a_d}\left[C\bar{S}^m(N_d-N_0)-\psi(a_d,a_0)\le 0\right]}\Big|(a_d=A) \tag{32}$$

The updated reliability can be employed to make decisions about what to do after the NDI. Possible alternatives include doing nothing, rescheduling the next inspection to an earlier date, or repairing/replacing the damaged element. If the updated reliability is considerably higher than the target reliability, the bogies can be seen as safety.

4. Case study

The previous sections discussed and quantified the proposed methodology. A practical fatigue analysis problem for bogies is employed to illustrate the advantages and efficiency of the proposed approach. For simplicity, we only consider two comprehensive models, i.e., Walker and Foreman (Paris 1964, Foreman 1967, Lv *et al.* 2015).

Bogies are one of the main parts of trains, which carry both static loads due to the body weight, and dynamic loads resulting from the rail surface roughness and imperfect wheels. Bogie frames are always subjected to dynamic random loads and other fatigue phenomena. The bogie frame is adopted by welded structure, the main framework architecture is H-shaped in the horizontal plane, which is composed of two box-shaped side sills. The overall composition of the box beam welding, and the central concave belly of the fish box structure composed of a spring seat side beam welding, basic brake mounts, anti-roll torsion bar seat, etc., the cavity has a thickness of 10mm stiffener plate. Box beam structure for the central opening, and the transverse beam welding has ended with stopper seat, traction rod seat, motor bracket, gearbox bracket and secondary lateral damper seat and so on.

The load history was obtained from strain measurements on a bogie frame. A three-dimensional finite element model

Table 1 Synthesis of the results mean stress/dynamic stress amplitude on the bogies

Part name	No.	Location	Average stress	Dynamic stress amplitude	Materials area
Beam and side beams connecting area	1	Within a support beam and side sill beam weld connection	47.4	64.3	Weld
	2	Beams and side beams connecting welds	66.4	51.6	Weld
	3	Cover plate with the support of the beam connecting the beams and side beams under three side beams connecting welds Department	116.1	58.5	Weld
Side sill area	4	Positioning seat upright plate portion of the opening arc bends	28.4	60.1	Base metal
	5	Positioning seat cover is connected with the lower side beam welds	67.6	47.6	Weld
	6	Under positioning seat cover parts connected with the vertical plate welds	76.1	53.5	Weld
	7	Anti-snake-seat legislature damper plate	0	51.0	Base metal
	8	Anti-snake damper seat and side sill outer webs connecting portion	33.8	28.4	Base metal
Beam area	9	Brake bracket vertical plate	0	55.8	Base metal
	10	Brake bracket and beam connection area	25.4	37.4	Weld
	11	Anti-roll torsion bar seat ribs	0	78.1	Base metal
	12	Longitudinal beams and beam weld connection	41.8	79.5	Weld
	13	Gearbox boom stand upright plate	0	60.3	Base metal

of a simplified bogie frame was developed for stress analysis, as shown in Fig. 1. According to the framework structure and analysis of static strength, fatigue crack tends to happen on 13 major parts that endure larger stress. Table 1 shows the calculation results of mean stress and dynamic stress amplitude in critical stress areas. We can find that the most critical area is the welded joint of longitudinal beams and beams, with the maximum stress is 79.5MPa. Therefore, we should focus on the study of welded joint of longitudinal beams and beams.

To perform reliability analysis of welded joint of longitudinal beams and beams, the uncertainty of the basic random variables in the reliability limit state function associated with the crack models must be quantified. Since the crack growth rate is very high near the critical crack size, the effect of the critical size a_c on the fatigue is rather small compared with other random variables involved in the entire fatigue damage process.

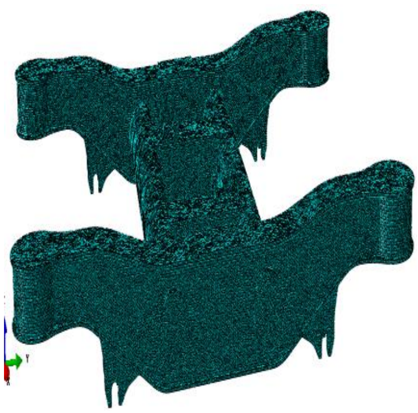

Fig. 1 The FEM model of the bogie frame

Therefore, in this study a_c can be considered as a deterministic parameter for simplicity. For illustrative purposes, a_c is considered to be 0.8mm for welded joint of longitudinal beams and beams. Then the initial crack size, geometry parameter, load process and material properties are shown in Table 2.

Suppose that the welded joint of longitudinal beams and beams has been inspected at about N=500,000 and three inspection results are considered respectively: no crack detection; crack size a_d =0.15 mm is detected; crack size a_d =0.35 mm is detected. According to Eqs. (29) and (32), the model weights and failure probability corresponding to the number of cycles are updated. The updated failure probabilities for the three cases are also plotted in Fig. 2, along with prior failure probability.

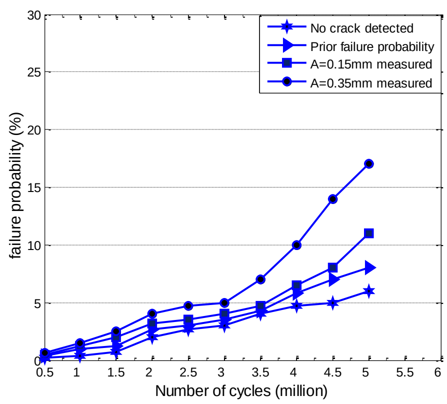

Fig. 2 Update failure probability with different inspection results

Table 2 Statistical characteristics of variables

Variable	Distribution	Mean	COV
C	Lognormal	1.556×10^{-6}	0.59
n	Normal	3.15	0.12
m	Normal	2.971	0.016
a_c	Constant	0.8mm	_
a_1	Lognormal	0.035mm	0.008
ΔK_{th}	Normal	$354\,\text{MPa}\sqrt{\text{mm}}$	24.6
K_C	Normal	$1934\,\text{MPa}\sqrt{\text{mm}}$	124.3
S	Normal	41.8 MPa	27.5

Table 3 Update of model weight after inspection

	Before inspection	No crack detected	a_d =0.15 mm	a_d =0.35 mm
P(M1) (Walker)	0.5	0.5	0.432	0.391
P(M2) (Foreman)	0.5	0.5	0.568	0.609

It can be observed that the updated failure probability is smaller than the prior one in the case of no crack detection, which means that the structure is more reliable than estimated previously. The failure probability increases when a crack is detected. The larger the detected crack is, the higher the probability of structural failure is.

Table 3 shows the update of model weights through inspection. When no crack is detected, model weights almost have no change. It also indicates that the weight of the Foreman model increases with the growth of the measured crack. This is also an expected result since comparatively the walker model is more conservative than the Foreman model.

5. Conclusions

In this paper, a Bayesian approach is proven to have the potential to provide more accurate estimations by combining evidence, such as test data with prior knowledge available. Several sources of uncertainty-physical variability, data uncertainty and modeling uncertainty are included in fatigue reliability analysis. The Bayesian model is proposed to involve various uncertainties in predicting fatigue damage of bogies, which combines multiple physical models and statistical models, and gives a more reasonable, comprehensive prediction of reliability. Bogies fatigue is a complex phenomenon affected by many uncertainties. A Bayesian model is presented to consider various uncertainties in predicting fatigue damage for bogies. With the application of modern nondestructive inspection techniques, the fatigue flaws can be detected for bogie structures, and fatigue reliability can be updated by Bayesian theorem with inspection data. Three contributions in the paper are:

(1) The analyses on the crack growth models and sources of uncertainty for bogies are given.

(2) A Bayesian model is used to account for uncertainty of various sources in predicting fatigue damage of structural components.

(3) The Bayesian inference is explained in the view of updating distribution of fatigue life using inspection data.

Acknowledgments

This research was supported by the National Natural Science Foundation of China under the contract number 51775090.

References

Aid, A., Amrouche, A., Bouiadjra, B.B., Benguediab, M. and Mesmacque, G. (2011), “Fatigue life prediction under variable loading based on a new damage model”, *Mater. Des.*, **32**(1), 183-191.

Chiachío, J., Chiachío, M., Saxena, A., Sankararaman, S., Rus, G. and Goebel, K. (2015), “Bayesian model selection and parameter estimation for fatigue damage progression models in composites”, *Int. J. Fatigue*, **70**, 361-373.

Cross, R., Makeev, A. and Armanios, E. (2007), “Simultaneous uncertainty quantification of fracture mechanics based life prediction model parameters”, *Int. J. Fatigue*, **29**(8), 1510-1515.

Dong, P. (2001), “A structural stress definition and numerical implementation for fatigue analysis of welded joints”, *Int. J. Fatigue*, **23**(10), 865-876.

Edwards, G. (1984), “A bayesian procedure for drawing inference from random data”, *Reliab. Eng.*, **9**(1), 1-17.

Fatemi, A. and Yang, L. (1998), “Cumulative fatigue damage and life prediction theories: a survey of the state of the art for homogenous materials”, *Int. J. Fatigue*, **20**(1): 9-34.

Foreman, R.G., Keart, V.E. and Engle, R.M. (1967), “Numerical analysis of crack propagation in cycle-loaded structures”, *J. Basic Eng.*, **89**(3), 459-464.

Gelman, A., Carlin, J.B., Stern, H.S., Dunson, D.B., Vehtari, A. and Rubin, D.B. (2014), “*Bayesian Data Analysis*”, CRC press, Boca Raton, FL, USA.

Ghodrati, A., Seyed, R. and Bagheri, A. (2011), “Damage detection in plates based on pattern search and genetic algorithms,” *Smart Struct. Syst.*, **7**(2), 117-132.

Guo, J., Li Z. and Pecht, M. (2015), “*A Bayesian approach for Li-Ion battery capacity fade modeling and cycles to failure prognostics*”, *J. Power Sources*, **281**, 173-184.

Huang, H. and Yang, J.N. (2008), “Damage identification of substructure for local health monitoring”. *Smart Struct. Syst.*, **4**(6), 795-807.

Huang, H.Z., Cui, P., Peng, W., Gao, H. and Wang, H.K. (2014), “Fatigue lifetime assessment of aircraft engine disc via multi-source information fusion”, *Int. J. Turbo Jet Eng.*, **31**(2), 167-174.

Huang, H.Z., Huang, C.G., Peng, Z., Li, Y.F. and Yin, H. (2017), “Fatigue life prediction of fan blade using nominal stress method and cumulative fatigue damage theory”, *Int. J. Turbo Jet Eng.*, DOI: https://doi.org/10.1515/tjj-2017-0015.

Li, X.Y., Huang, H.Z., Li, Y.F. and Zio, E. (2018), “Reliability assessment of multi-state phased mission system with non-repairable multi-state components”, *Appl. Math. Model.*, **61**, 181-199.

Li, Y.F., Zhu, S.P., Li, J., Peng, W. and Huang, H.Z. (2015), “Uncertainty analysis in fatigue life prediction of gas turbine blades using Bayesian inference”, *Int. J. Turbo Jet Eng.*, **32**(4), 319-324.

Li, Z., Deng, Y. and Mastrangelo, C. (2015), “*Model selection for degradation-based Bayesian reliability analysis*”, *J. Manuf.*

Syst., **37**, 72-82.

Liu, Y. and Mahadevan, S. (2005), “Multiaxial high-cycle fatigue criterion and life prediction for metals”, *Int. J. Fatigue*, **27**(7), 790-800.

Luo, J. and Bowen, P. (2003), “A probabilistic methodology for fatigue life prediction”, *Acta. Mater.*, **51**(12), 3537-3550.

Lv, Z., Huang, H.Z., Gao, H, Zuo, F.J. and Wang, H.K. (2015), “Lifetime prediction for turbine discs based on a modified Walker strain model”, *J. Mech. Sci. Techno.*, **29**(10), 4143-4152.

Makeev, A., Nikishkov, Y. and Armanios, E. (2007), “A concept for quantifying equivalent initial flaw size distributions in fracture mechanics based life prediction models”, *Int. J. Fatigue*, **29**(1), 141-145.

Mahadevan, S. and Rebba, R. (2006), “Validation of reliability computational models using Bayes networks”, *Reliab. Eng. Syst. Safe.*, **87**(2), 223-232.

Mi, J., Li, Y.F., Peng, W. and Huang, H.Z. (2018), “Reliability analysis of complex multi-state system with common cause failure based on evidential networks”, *Reliab. Eng. Syst. Safe.*, **174**, 71-81.

Paris, P.C. (1964), “The fracture mechanics approach to fatigue”, *New York: Syracuse University Press*, 107-132.

Rebba, R. (2005), “*Model validation and design under uncertainty*”, PhD dissertation, Vanderbilt University, Nashville.

Richard, C. and Andrew, M. (2007), “Simultaneous uncertainty quantification of fracture mechanics based life prediction model parameters”, *Int. J. Fatigue*, **29**(8), 1510-1515.

Sankararaman, S., Ling, Y., Shantz, C. and Mahadevan, S. (2009), “*Uncertainty quantification in fatigue damage prognosis*”, The proceedings of the 1st annual conference of the Prognostics and Health Management Society, San Diego, CA. September 27-October 1.

Sankararaman, S., Ling Y. and Mahadevan, S. (2010), “Statistical inference of equivalent initial flaw size with complicated geometry and multi-axial loading”, *Int. J. Fatigue*, **32**(10), 1689-1700.

Sankararaman, S., Ling, Y., Shantz, C. and Mahadevan, S. (2011), “Inference of equivalent initial flaw size under multiple sources of uncertainty”, *Int. J. Fatigue*, **33**(2), 75-89.

Soares, C. G. and Garbatov, Y. (1999), “Reliability of maintained, corrosion protected plates subjected to non-linear corrosion and compressive loads”, *Mar. Struct.*, **12**(6), 425-445.

Weertman, J. (1966), “Rate of growth of fatigue cracks calculated from the theory of infinitesimal dislocations distributed on a plane”, *Int. J. Frac. Mech.*, **2**(2), 460-467.

Xiang, J., Matsumoto, T., Long, J., Wang, Y. and Jiang, Z. (2012), “A simple method to detect cracks in beam-like structures”, *Smart Struct. Syst.*, **9**(4), 335-353.

Zheng, B., Huang, H.Z., Guo, W., Li, Y.F. and Mi, J. (2018), “Fault diagnosis method based on supervised particle swarm optimization classification algorithm”, *Intell. Data Anal.*, **22**(1), 191-210.

Zhang, R.X. and Mahadevan, S. (2000), “Model uncertainty and Bayesian updating in reliability-based inspection”, *Struct. Saf.*, **22**(2), 145-160.

Zhao, Z., Haldar, A. and Breen, Jr. (1994), “Fatigue reliability updating through inspection of steel bridges”, *J. Struct. Eng.*, **120**(5), 1624-1641.

Zheng, R. and Ellingwood, B. (1998), “Role of non-destructive evaluation in time-dependent reliability analysis”, *Struct. Saf.*, **20**(4), 303-323.

Zhou, J., Huang, H.Z. and Peng, Z. (2017), “Fatigue life prediction of turbine blades based on modified equivalent strain model”, *J. Mech. Sci. Technol.*, **31**(9), 4203-4213