Evidence Relationship Matrix and Its Application to D-S Evidence Theory for Information Fusion

Xianfeng Fan, Hong-Zhong Huang, and Qiang Miao

School of Mechatronics Engineering,
University of Electronic Science and Technology of China,
Chengdu, Sichuan, 610054, China
fanxf@yahoo.com, hzhuang@uestc.edu.cn, mqiang@uestc.edu.cn

Abstract. D-S evidence theory has been studied and used for information fusion for a while. Though D-S evidence theory can deal with uncertainty reasoning from imprecise and uncertain information by combining cumulative evidences for changing prior opinions using new evidences. False evidence generated by any fault sensor will result in evidence conflict increasing and inaccurate fused results. Evidence relationship matrix proposed in this paper depicts the relationship among evidences. False evidences can be identified through the analysis of relationships among evidences. Basic probability assignments related to the false evidences may be decreased accordingly. The accuracy of information fusion may be improved. Case studies show the effectiveness of the proposed method.

Keywords: evidence relationship matrix, D-S evidence theory, and information fusion.

1 Introduction

The Dempster-Shafer (D-S) evidence theory was proposed by Shafer, who built upon Dempster's research [1]. It is able to calculate probabilities that evidence supports the propositions and offers an alternative approach to dealing with uncertainty reasoning from imprecise and uncertain information. The theory is suitable to taking into account the disparity of knowledge types due to the fact that it is able to provide a federative framework, and combine cumulative evidences for changing prior opinions in the light of new evidences [1, 2]. Therefore, the study and application of D-S evidence theory for information fusion attract researchers interests [3, 4, 5].

We still face some challenges during using the theory in practice. For example, evidence may not be sufficient to support the basic probability assignment because of the measurement errors incurred by sensors. Recently, fuzzy theory was introduced to modify the basic probability assignment (BPA) with the consideration of evidence sufficiency [6, 7]. But, false evidences are not considered. If any sensor has fault, the acquired data is not correct any more. False evidences will occur. The conflicts among evidences may be bigger than before. Wrong fusion results may arise. In this paper, evidence relationship matrix that can reflect the relationships among evidences is proposed. Through studying the evidence relationship, we can identify false

E. Corchado et al. (Eds.): IDEAL 2006, LNCS 4224, pp. 1367 – 1373, 2006.

evidences. The BPAs related to the false evidences can be decreased greatly. Therefore, the accuracy of information fusion through D-S evidence theory can be improved accordingly.

The rest of the paper is organized as follows. D-S evidence theory is illustrated briefly in Section 2. Evidence relationship matrix is proposed in Section 3. The modification of BPAs based on evidence relationship matrix is investigated. An example is given out to validate the proposed method in Section 4. Conclusions and discussion are presented in the last section.

2 D-S Evidence Theory

Let Θ be a finite nonempty set of mutually exclusive alternatives, and be called discernment frame containing every possible hypothesis.

Basic probability assignment is a function, $m: 2^{\Theta} \rightarrow [0,1]$, such that $m(\varnothing)=0$ where $\varnothing$ denotes an empty set, and $\sum m(X)=1$ for any $X \subseteq \Theta$. The power set 2^{Θ} is the set of all the subsets of Θ including itself [1]. Given a piece of evidence, a belief level between [0, 1], denoted by $m(\cdot)$, is assigned to each subset of Θ. Each subset contains one or more hypothesis. If a feature for a hypothesis exists, the corresponding BPA is said to be fired by the feature and the feature is called evidence. This BPA will be involved in information fusion. Otherwise, the BPA for the hypothesis will not be fired and considered for information fusion.

The total belief level committed to X, $Bel: 2^{\Theta} \rightarrow [0,1]$, is obtained by calculating the belief function for X as Eq. (1). $Bel(X)$ represents the belief level that a proposition lies in X or any subset of X.

$$Bel(X) = \sum_{Y \subseteq X, X \subseteq \Theta} m(Y). \tag{1}$$

The plausibility function defined below measures the extent, to which we fail to disbelieve the hypothesis of X, $Pl: 2^{\Theta} \rightarrow [0,1]$.

$$Pl(X) = \sum_{Y \cap X \neq \varnothing, X, Y \subseteq \Theta} m(Y). \tag{2}$$

Both imprecision and uncertainty can be represented by Bel and Pl. The relationship between them is

$$\begin{cases} Pl(A) = 1 - Bel(\overline{A}) \\ Pl(A) \geq Bel(A) \end{cases}. \tag{3}$$

Where, $\overline{A}$ is the negation of hypothesis A. $Bel(A)$ and $Pl(A)$ are the lower limit and the upper limit of belief level of A, respectively. $Pl(A)$-$Bel(A)$ for $A \subseteq \Theta$ represents the ignorance level in hypothesis A.

Multiple evidences can be fused using Dempster's combination rule, shown as Eq. (4) [1]. Evidences of any subsets X and Y of Θ can be used to calculate the belief level in a new hypothesis C. $C = X \cap Y$. If $C = \varnothing$, it means evidences conflict with each other totally and the belief level in hypothesis C is then null.

$$m(C) = m_i(X) \oplus m_{i'}(Y) = \begin{cases} 0, & If \quad X \cap Y = \varnothing \\ \dfrac{\sum_{X \cap Y = C, \forall X, Y \subseteq \Theta} m_i(X) \times m_{i'}(Y)}{1 - \sum_{X \cap Y = \varnothing, \forall X, Y \subseteq \Theta} m_i(X) \times m_{i'}(Y)}, & If \quad X \cap Y \neq \varnothing \end{cases} \tag{4}$$

where i (i') denotes the i^{th} (i'^{th}) evidence. $m_i(X)$ and $m_{i'}(Y)$ are the BPA of X supported by evidence i and the BPA of Y supported by evidence i', respectively. Let

$$k_{i,i'} = \sum_{X \cap Y = \varnothing} m_i(X) \times m_{i'}(Y), \tag{5}$$

$k_{i,i'}$ is called the conflict factor between the two evidences i and i', where $k_{i,i'} \neq 1$.

Dempster's combination rule can be generalized to more than two hypotheses, as Eq. (6). The final result represents the synthetic effects of all evidences.

$$m = m_1 \oplus m_2 \oplus \cdots \oplus \cdots = (((m_1 \oplus m_2) \oplus \cdots) \oplus \cdots). \tag{6}$$

In practice, because the collected data from sensors have errors and the features extracted may not be sufficient, we may have no adequate evidence to support a certain hypothesis. Fan and Zuo solve the issue through the introduction of fuzzy theory [6]. For evidence e_i, the evidence sufficiency can be realized by the attenuation of BPA using sufficiency index μ_i, as Eq. (7). The new BPA is denoted by $m_{i,*}(\cdot)$.

$$m_{i,*}(A) = \begin{cases} \mu_i \cdot m_i(A), & A \subset \Theta \\ 1 - \sum_{B \subset \Theta} \mu_i \cdot m_i(B), & B \subset \Theta, A = \Theta \end{cases} \tag{7}$$

where $i = 1,2,\cdots,n$, n is the number of evidences, and * denotes that the BPA has incorporated evidence sufficiency. All the above analysis is based on the assumption that all sensors are normal. In practice, we may collect false data and obtain false evidence if any unknown fault sensor exists. For this case, information fusion results are suspect and may be wrong.

3 The Evidence Relationship Matrix and the Modification of BPAs

Because all the information comes from a same system usually, there may be a relationship between any two evidences. $\boldsymbol{E}$ is used to represent evidence set in this paper. $\boldsymbol{E} = \{e_i \mid i = 1,2,\cdots,n\}$. n is the number of all the evidences that may be obtained for information fusion. $r_{i,j}$ represents the relationship between evidences e_i and e_j. $j = 1,2,\cdots,n$. It means that if evidences e_i appear, the appearance probability of evidence e_j should be $r_{i,j}$. $r_{i,j} \in [0,1]$. If there is no relationship between evidences e_i and e_j, $r_{i,j} = 0.5$. It means that if e_i appears, the appearance possibility of e_j is 50%. The meaning of $r_{i,j} < 0.5$ is that if e_i appears, the appearance possibility of e_j is less than 50%. For evidence self, $r_{i,i} = 1$. In this paper, $r_{i,j}$ are determined through system analysis. For example, machine faults are usually reflected through vibration, acoustic, wear debris, oil temperature, electrical current, and function performance. The pattern identification accuracy of machine faults is greatly depended on the above multi signatures. The determination of $r_{i,j}$ is based on the following relationships. Such as: if the vibration increases, acoustic level will increase with the possibility of 80%. If there are lots of metal particles in oil debris, vibration will increase with the possibility of 90%. These possibilities $r_{i,j}$ can be obtained by experts experience and statistic based on past fault modes. Then, we may have the following matrix, called evidence relationship matrix.

$$R=\begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,n} \\ r_{2,1} & r_{2,2} & \cdots & r_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n,1} & r_{n,2} & \cdots & r_{n,n} \end{bmatrix}=\begin{bmatrix} 1 & r_{1,2} & \cdots & r_{1,n} \\ r_{2,1} & 1 & \cdots & r_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n,1} & r_{n,2} & \cdots & 1 \end{bmatrix}. \tag{8}$$

In this matrix, for evidence e_i, when the number of j that satisfies $r_{j,i}>0.5$ is more than one, it means that more than one evidences support the appearance of e_i. In other words, the appearance possibility of e_i becomes bigger. For each row in Eq. (8), we can obtain the appearance possibility of e_i, P_i^R, from total other evidences by

$$P_i^R=\sum_j r_{i,j}. \tag{9}$$

Accordingly, for each column in Eq. (8), we can obtain the appearance possibility of e_i, P_i^C, from total other evidences by

$$P_j^C=\sum_i r_{i,j}. \tag{10}$$

Because $i,j=1,2,\cdots,n$, we may write P_j^R as P_i^R. At last, we can reach the general appearance possibility of an evidence i by

$$P_i=(P_i^R+P_i^C)/2\,. \tag{11}$$

The modification of BPAs can be performed. Firstly, construct the matrix in Eq. (8). Secondly, calculate P_i using Eqs. (8)-(11). Thirdly, define an appearance possibility index $\hat{P}_i$. $\hat{P}_i=P_i/\max(P_i)$. At last, we obtain the modified BPAs $m_i^*(X)=\hat{P}_i m_i(X)$. $X\subset\Theta$. In addition, considering sensor error, we introduce synthetical index ξ_i. $\xi_i=\hat{P}_i\cdot\mu_i$. μ_i is evidence sufficiency index. The details can be referred to [6]. Therefore, the modified BPAs can be obtained by

$$\begin{cases} m_i^*(X)=\xi_i m_i(X) \\ m_i^*(\Theta)=1-\sum m_i^*(X) \end{cases} \quad \text{for } X\subset\Theta. \tag{12}$$

In Eq. (12), we assume that all the evidences have same importance in order to focus on the issue mentioned in this paper.

In practice, it is impossible that all evidences appear at a same time. Only the evidences related to a hypothesis, which is true, appear. Therefore, we have the practical relationship matrix $\hat{\boldsymbol{R}}$. The dimension of $\hat{\boldsymbol{R}}$ is less than that of $\boldsymbol{R}$.

At last, we substitute the $m_i(X)$ and $m_i(Y)$ by $m_i^*(X)$ and $m_i^*(Y)$, respectively. Using Eq. (4), we can fuse information and greatly avoid the affect of false evidences.

4 Case Studies

In order to verify the proposed method, an example is studied. Suppose $\Theta = \{F_1, F_2, F_3\}$, $\boldsymbol{E} = \{e_i \mid i = 1,2,3\}$. The BPAs are shown in Table 1.

Table 1. The basic possibility assignments

	$m(\{F_1\})$	$m(\{F_2\})$	$m(\{F_3\})$	$m(\Theta)$
e_1	0.1	0.2	0.7	0
e_2	0.4	0.5	0	0.1
e_3	0.8	0.1	0.1	0

In this example, we assume that all the evidences are sufficient and have the same importance for information fusion. Therefore, $\mu_i = 1$. In Table 1, we find that evidences e_1 support $\{F_3\}$ greatly. While, evidences e_2 and e_3 do not support $\{F_3\}$ very well. Obviously, both of evidences e_2 and e_3 conflict with evidence e_1. Therefore, e_1 may be false evidence.

If we do not consider the phenomena discussed above, according to Eq. (4), we fuse the evidence $\{e_i \mid i = 1,2,3\}$ and obtain the following results. $m(\{F_1\}) = 0.6778$, $m(\{F_2\}) = 0.2035$, $m(\{F_3\}) = 0.1187$. Correspondingly, belief level and plausibility can be obtained using Eqs. (1) and (2), respectively. Such as, $Bel(\{F_1\}) = 0.6778$, $Pl(\{F_1\}) = 0.6778$.

We then consider the relationship among evidences in order to embody the conflict issue mentioned above. The evidence relationship matrix is shown in Eq. (13). Based on Eqs. (8)-(11), we are able to calculate $\hat{P}_i$. $\hat{P}_i$ are equal to 0.6667, 0.911, and 1, respectively, when i is equal to 1, 2 and 3. Based on Eq. (12), Table 1 is modified to Table 2.

$$R = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \end{array} \begin{array}{c} \begin{array}{ccc} e_1 & e_2 & e_3 \end{array} \\ \begin{bmatrix} 1 & 0.1 & 0.6 \\ 0.2 & 1 & 0.9 \\ 0.1 & 0.9 & 1 \end{bmatrix} \end{array}. \tag{13}$$

Table 2. The modified basic possibility assignments

	$m^*(\{F_1\})$	$m^*(\{F_2\})$	$m^*(\{F_3\})$	$m^*(\Theta)$
e_1	0.0667	0.1333	0.4667	0.333
e_2	0.3644	0.4544	0	0.1801
e_3	0.8	0.1	0.1	0

According to Eq. (4) and Table 2, we fuse the evidence $\{e_i \mid i = 1,2,3\}$ again and obtain the following results. $m(\{F_1\}) = 0.7960$ · $m(\{F_2\}) = 0.1368 \cdot m(\{F_3\}) = 0.0672$ · Correspondingly, belief level and plausibility can be obtained using Eqs. (1) and (2), respectively. Such as, $Bel(\{F_1\}) = 0.7960$ · $Pl(\{F_1\}) = 0.7960$ ·

Compared with the fusion results without introduction of evidence relationship matrix, the BPA of $\{F_1\}$ increases using the proposed method. The comparisons are shown in Fig. 1. According to Eqs. (1) and (2), the *Bel* and *Pl* of $\{F_1\}$ will increase using the proposed method, correspondingly. The BPA, *Bel* and *Pl* of $\{F_3\}$ decrease greatly. These results show that the role of evidence e_1 for fusion is decreased greatly.

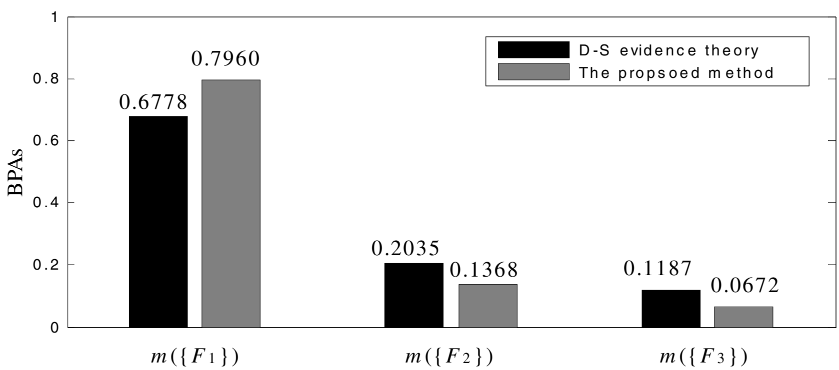

Fig. 1. The comparison between D-S evidence theory and the proposed method

5 Conclusions and Discussions

In this study, relationships among evidences are considered in D-S evidence theory for information fusion. Evidence relationship matrix is proposed to depict these relationships. Appearance possibility index obtained using the evidence relationship matrix is proposed as well. Case studies show that the proposal of evidence relationship matrix and appearance possibility index can help to decrease the role of false evidences for information fusion. The credibility of information fusion is improved compared with the information fusion using traditional D-S evidence theory. The evaluation of relationships between any two evidences needs study further.

Acknowledgments

The work reported in this paper was partially supported by the Open Fund of the State Key Laboratory of Vibration, Shock and Noise of Shanghai Jiaotong University under grant VSN-2006-03.

References

[1] Shafer, G.: A Mathematical Theory of Evidence, Princeton, NJ: Princeton University Press, (1976).

[2] Fabre, S., Appriou A., Briottet X.: Presentation and Description of Two Classification Methods using Data Fusion based on Sensor Management. Information Fusion. 2 (2001) 49-71

[3] Rottensteiner F., Trinder J., Clode S., Kubik K.: Using the Dempster - Shafer Method for the Fusion of LIDAR Data and Multi - Spectral Images for Building Detection. Information Fusion 6 (2005) 283-300

[4] Basir O., Karray F., Zhu H.: Connectionist-Based Dempster–Shafer Evidential Reasoning for Data Fusion. IEEE Transactions on Neural Networks. 6(2005) 1513-1530

[5] Fan X., Huang H., Miao Q.: Agent-based Diagnosis of Slurry Pumps Using Information Fusion. Proceedings of the International Conference on Sensing, Computing and Automation. ICSCA 2006. Chongqing, China. May 8-11 (2006) 1687-1691

[6] Fan X., Zuo, M. J.: Fault Diagnosis of Machines based on D-S Evidence Theory. Part 1: D-S Evidence Theory and Its Improvement. Pattern Recognition Letters. 5(2006) 366-376

[7] Huang H. Z.: Fuzzy Multi-Objective Optimization Decision-Making of Reliability of Series System. Microelectronics and Reliability. 3 (1997) 447- 449