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Bayesian degradation assessment of CNC machine tools considering unit non-homogeneity †

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Abstract

Field reliability assessment and prediction is critical for the estimation, operation and health management of CNC machine tools. The classical methods for field reliability of CNC Machine Tools assessment and prediction are challenged with the issues of expensive reliability tests, small sample size and unit non-homogeneity. In order to solve these problems, this paper introduces a degradation analysis based reliability assessment method for CNC machine tools under performance testing. Since the degradation is an independent increment process, the gamma process is employed to characterize the degradation process of CNC machine tools. The random effects are introduced to accommodate performance degradation model with unit non-homogeneity. The parameters of model are updated by Bayesian estimation approach. As a case study, the CNC Machine Tools is studied to illustrate the approach. And the proposed method is demonstrated precise for practical use.

Keywords: CNC machine tools; Performance degradation; Gamma process; Unit non-homogeneity; Bayesian method

1. Introduction

The CNC machine tools are complex electromechanical systems. As the basic production equipment in manufacturing industry, it has been widely used in many business sectors, such as aerospace, rail transit, navigation and military companies [1]. The failure of CNC machine tools may result in malfunctioning equipment of these engineering systems and lead to economic loss. Therefore, reliability assessment of CNC machine tools is an important requirement of industry and academia [2-4]. For the long-life and high-reliability products such as the CNC machine tools, traditional reliability analysis methods based on failure data are inaccurate due to the insufficient analysis data. To address this problem, the reliability of CNC machine tools is assessed by the method based on degradation analysis in this paper. In general, degradation is a natural phenomenon of any engineering system, and it is the reduction in performance and reliability. Finally, degradation results in a failure once the accumulated damage reaches or exceeds a predefined threshold [5-7]. Degradation models are widely used in the field of reliability engineering, specific discussion and related applications can be found in Nelson [8], Meeker [9], Endrenyi [10]. The machining accuracy, which is very critical to the CNC machine tools, is selected as the performance indicator to do the degradation analysis in this paper. The degree of accuracy that the machined parts can achieve is heavily dependent on the machining accuracy of the CNC machine tools. Moreover, it is one of the key indicators in acceptance check of the CNC machine tools. And the CNC machine tools fail if the machining accuracy degradation reaches the failure threshold.

Among various types of degradation models, gamma process is a stochastic process to characterize the degradation process that involves independent and non-negative increments. It has been discussed by many researchers, such as Noortwijk [11] and Bagdonavičius [12]. Due to design tolerances, manufacturing variation and other uncertainties, reliability of the same type of CNC machine tools may have inherent difference called unit non-homogeneity. In order to get a more accurate reliability assessment results, it is necessary to incorporate the unit non-homogeneity of products reliability into the degradation assessment model. This measure has theoretical value and practical engineering significance, which has been confirmed by many studies [13, 14]. The random effects are introduced into the gamma process model to describe the unit non-homogeneity [15]. Based on these studies, unit non-homogeneity is introduced to the CNC machine tools machining accuracy degradation model and the detailed

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analysis is implemented using Bayesian method in this paper.

In the Bayesian reliability method, the statistical model consists of the prior distribution and the likelihood function. The likelihood function is obtained from the sampling distribution of the test data and it is expressed as the probability density function (PDF) of the observed test data. According to general experiences, a prior distribution is the distribution of random variables that should be satisfied before the experiment is observed [16, 17]. The prior distributions are classified into informative prior distribution and non-informative prior distribution. Generally, the informative prior comes from historical data, literature, or expert opinion. And the uniform distribution is usually used to describe a non-informative prior distribution [18]. In this paper, the reliability assessments using the two kinds of prior distributions are carried out separately and the differences are compared and discussed.

2. The degradation models

The Wiener process and gamma process are frequently-used stochastic processes for characterizing the degradation processes. The increments in the gamma process must be positive while the negative increments is allowed in the Wiener process. Gamma process could be considered as an approximate process of the compound Poisson process, which is consistent with the physical mechanism of the actual performance evolvement of the CNC machine tools. The performance degradation of the machine tools is often caused by the cutting force and the external factors. Therefore, the gamma process that contains independent non-negative increments, is used to describe the machining accuracy degradation of the CNC machine tools.

The gamma process is a stochastic processes $\{X(t), t>0\}$ with the following properties:

(1) $X(0)=0$;

(2) $X(t)$ is independent incremental process;

(3) Increments obey gamma distribution: $X(t+\Delta t)-X(t) \sim Ga(\eta(t+\Delta t)-\eta(t), \lambda)$.

where $\eta(t)$ is the shape parameter and a right-continuous non-decreasing process in [0, ∞), $\eta(0)=0$. λ is the scale parameter and $\lambda>0$.

The probability density function (PDF) $g(x)$ of the gamma process is defined by,

$$g_{X(t)}(x|\eta(t),\lambda)=\frac{\lambda^{\eta(t)}}{\Gamma(\eta(t))}x^{\eta(t)-1}\exp(-\lambda x)I_{X(0,\infty)}(x) \tag{1}$$

where

$$I_{(0,\infty)}(x)=\begin{cases}1 & x\in(0,\infty)\\ 0 & x\notin(0,\infty)\end{cases} \tag{2}$$

and $\Gamma(\eta)=\int_0^\infty x^{\eta-1}e^{-x}dx$ is gamma function.

The failure threshold is defined as C, and the lifetime T of the product or system is defined as the time at which the degradation value reaches the degradation failure threshold for the first time, shown as,

$$T=\inf\{t|X(t)\geq C\}. \tag{3}$$

Then, the system reliability $R(t)$ at time t can be expressed as,

$$R(t)=\mathrm{P}_r\{\sup_{s\leq t}(X(t))\leq C\}. \tag{4}$$

Due to the monotone increasing features of the Gamma process, combining Eqs. (1) and (4), $R(t)$ can be expressed as

$$\begin{aligned}R(t)&=P_r\{T\geq t\}=P_r\{X(t)<C\}\\ &=\int_0^C g_{X(t)}(x|\eta(t),\lambda)dx\\ &=\frac{1}{\Gamma(\eta(\mathrm{t}))}\int_0^{C\lambda}u^{\eta(t)-1}e^{-\lambda}du.\end{aligned} \tag{5}$$

As mentioned in the Sec. 1, the same type of CNC machine tools may have inherent difference called unit non-homogeneity. The unit non-homogeneity of products is manifested as different CNC machine tools have their own unique rate of performance evolution in the same overall, and their performance increments have their own characteristics. To model the unit non-homogeneity in different individuals in a unified population, the random effects are introduced into the degradation model [13-15]. In many studies, the researchers assumed that the random effects in the product affected the scale parameters and had no effect on the shape parameters [15, 19]. When gamma process is used to describe performance degradation modeling of the same products of unit non-homogeneity, it indicates the same shape parameters, different scale parameters.

A stochastic process $\{X(t), t>0\}$ is a gamma process, whose shape parameter is $\eta(t)=\eta t$, and scale parameters is v^{-1}, where v is the random effect. The increments within Δt follow the gamma distribution: $\Delta X(t)\sim Ga(\eta\Delta t,\ v^{-1})$. In this equation, $\Delta X(t)=X(t+\Delta t)-X(t)$, $\eta\Delta t=\eta(t+\Delta t)-\eta(t)$. Lawless [15] defined that scale parameter v^{-1} of the unit non-homogeneity degradation process obeys a gamma distribution with shape parameter γ^{-1} and scale parameter δ: $v\sim Ga(\gamma^{-1},\delta)$. The marginal density of $X(t)$ follows as,

$$\begin{aligned}f(X)&=\int_0^\infty g(X|\eta\Delta t,v^{-1})\mathrm{g}(v^{-1}|\gamma^{-1},\delta)dv\\ &=B(\eta\Delta t,\delta)^{-1}\gamma^{\delta}X^{\eta\Delta t-1}/(X+\gamma)^{\eta\Delta t+\delta}\end{aligned} \tag{6}$$

where the $B(\eta\Delta t,\delta)=\Gamma(\eta\Delta t)\Gamma(\delta)/\Gamma(\eta\Delta t+\delta)$ is the beta function. Since it is strictly monotonically increasing in the time domain and the $\delta X(t)/(\gamma\eta t)$ obeys a F-distribution whose cumulative density function (CDF) could be expressed as $F_{2\eta n,2\delta}$ [19] and that $X(t)/(\gamma+X(t))$ follows the beta distribution $Be(\eta\Delta t,\delta)$.

If the degradation process is described by the gamma process, the product fails when the degradation reaches the failure threshold C, with a failure time of T_c. The system reliability $R(t)$ can be expressed as,

$$\begin{aligned} R(t) &= 1 - P(T_C \le t) = 1 - P(X(t) \ge C) \\ &= P(t < T_C) = P(X(t) < C) \\ &= F_{2\eta n, 2\delta}\left(\frac{\delta C}{\gamma \eta t}\right). \end{aligned} \tag{7}$$

3. Parameter estimation of degradation

Suppose there are N samples, the sample number is denoted by $i, (i = 1, 2, \ldots, N)$. Each sample is subjected to M time degradation measurements and its serial number is $j, (j = 1, \ldots, M)$. $D(t_{ij})$ is the degradation amount of the ith sample at the jth measurement time, and the time is defined as t_{ij}. $D(t_{i0})$ is the degradation increment of the ith sample, and is usually set to 0. Then, the degradation increment of the ith sample can be expressed as,

$$\Delta d_{ij} = D(t_{ij}) - D(t_{i,j-1}). \tag{8}$$

By the degradation model introduced above, the degradation increments Δd_{ij} obey a gamma distribution $Ga(\eta \Delta t_{ij}, v^{-1})$, and $\Delta t_{ij} = t_{ij} - t_{i,j-1}$. It is assumed that the scale parameters of the degradation process obey another gamma distribution: $v^{-1} \sim Ga(\gamma^{-1}, \delta)$. When the degradation observations are obtained as D, the likelihood function based on D is,

$$\begin{aligned} & L(D \mid \eta, \gamma^{-1}, \delta) \\ &= \prod_{i=1}^{N} g(v \mid \gamma^{-1}, \delta) \prod_{j=2}^{M} g(\Delta d_{ij} \mid \eta \Delta t_{ij}, v^{-1}) \\ &= \prod_{i=1}^{N} \frac{v^{\delta-1}\gamma^{\delta}}{\Gamma(\delta)} \exp(-\gamma v) \prod_{j=2}^{M} \frac{(\Delta d_{ij})^{\eta \Delta t_{ij} - 1} v^{\eta \Delta t_{ij}}}{\Gamma(\eta \Delta t_{ij})} \exp(-v \Delta d_{ij}) \end{aligned} \tag{9}$$

where $\Delta d_{ij} = D_{ij} - D_{i,j-1}$, $\Delta t_{ij} = t_{ij} - t_{i,j-1}$ and $g(\bullet)$ is the PDF of a gamma distribution.

Assume the prior information about the accuracy degradation of CNC machine tools is obtained and quantified the prior distributions as $\pi(\eta)$, $\pi(\gamma^{-1})$ and $\pi(\delta)$ for model parameters.

The shape parameters η is obtained by using the interval method [20]. Historical data and subjective information are quantified into the probability distributions $\pi(\gamma^{-1})$ and $\pi(\delta)$, the gamma distribution is chosen for these distributions as the characteristic distributions for the model with random effects [21-24].

Based on the Bayesian theory, the joint posterior distribution can be obtained as,

$$\begin{aligned} & p(\eta, \gamma^{-1}, \delta \mid D) \propto \pi(\eta)\pi(\gamma^{-1})\pi(\delta) L(D \mid \eta, \gamma^{-1}, \delta) \\ &= \pi(\eta, \gamma^{-1}, \delta) L(D \mid \eta, \gamma^{-1}, \delta) \end{aligned} \tag{10}$$

$$\begin{aligned} &= \pi(\eta, \gamma^{-1}, \delta) \prod_{i=1}^{N} \frac{v^{\delta-1}\gamma^{\delta}}{\Gamma(\delta)} \exp(-\gamma v) \\ & \prod_{j=2}^{M} \frac{(\Delta d_{ij})^{\eta \Delta t_{ij} - 1} v^{\eta \Delta t_{ij}}}{\Gamma(\eta \Delta t_{ij})} \exp(-v \Delta d_{ij}) \end{aligned}$$

where $p(\eta, \gamma^{-1}, \delta \mid D)$ is the joint posterior distribution for model parameters. It is a description of the blend of prior information and the measured accuracy degradation data.

The performance evolution predicted of the individual CNC machine tools at the future observation time Δt_{ij} is presented as,

$$\begin{aligned} & f(\Delta d_{ij} \mid D) \\ &= \int_{\eta, \delta, \gamma > 0} p(\eta, \gamma^{-1}, \delta \mid D) \mathrm{g}(\Delta d_{ij} \mid \eta \Delta t_{ij}, v^{-1}) \mathrm{g}(v^{-1} \mid \gamma^{-1}, \delta) d\eta d\delta d\gamma. \end{aligned} \tag{11}$$

Following Eqs. (9) and (10), the reliability based on the joint posterior distribution can be expressed as,

$$R(\Delta t_{ij} \mid D) = \int_{\eta, \delta, \gamma > 0} p(\eta, \gamma^{-1}, \delta \mid D) F_{2\eta n, 2\delta}\left(\frac{\delta C}{\gamma \eta t}\right) d\eta d\delta d\gamma. \tag{12}$$

Since the analytical expression of Eqs. (10) and (12) are difficult to obtain, the Markov chain Monte Carlo (MCMC) method is used to sample the joint posterior distribution. In this paper, the OpenBUGS [25] is used to enable the realization of the MCMC.

In order to ensure that the samples by the MCMC method are generated by the correct target distribution, the monitoring on the convergence of the algorithm is crucial. The trace plots are monitored to check the convergence of simulation parameters of MCMC method [26]. The trace plots are the plots of the generated values versus the iterations.

4. Illustrative example

The machining accuracy of five samples is monitored to obtain the degradation information about deterioration of the CNC machine tools. Since they can be measured only when the CNC machine tools are in off-state, the test times of the samples are different and independent. The accuracy degradation paths can be depicted in Fig. 1.

In order to realize the reliability assessment of CNC machine tools, the Bayesian method is used to analyze the degradation data. On account of obvious difference in the degradation paths as shown in Fig. 1, gamma process model with random effects is used to characterize the degradation process of each tools. The degradation increments $\Delta d_{ij} = D(t_{ij}) - D(t_{i,j-1})$ with $i = 1, \ldots, 5$ and $j = 1, \ldots, 20$ obey the gamma distribution $Ga(\eta \Delta t_{ij}, v^{-1})$ with $v \sim Ga(\gamma^{-1}, \delta)$.

According to many existing studies [18, 27], if the prior distribution are non-informative priors, its parameters can be expressed as follows,

Table 1. Estimations of model parameters non-informative prior distribution.

	Mean	Standard deviation	Confidence interval	
			2.5 %	97.5 %
η	0.2334	0.03341	0.173	0.3042
δ	12.71	10.1	1.371	38.38
γ	11.75	9.519	0.9207	36.64

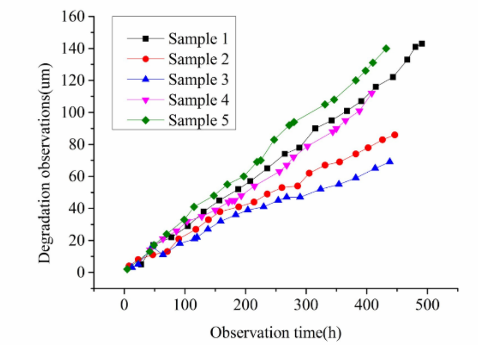

Fig. 1. Machining accuracy degradation paths.

$$\delta \sim U(0,10000), \gamma \sim U(0,10000), \eta \sim U(0,10000)$$

where $U(0, 10000)$ is the uniform distribution with interval $(0,10000)$.

In this paper, the uniform distribution is used as a kind of non-informative prior distribution to analyze the performance degradation data. On the one hand, it is limited by available prior information, and on the other hand, the analysis results can be consistent with the information contained in the performance degradation data. The posterior distribution of the model parameters is obtained by the MCMC method. The OpenBUGS is adopted to implement the posterior distribution of the model parameters based on the MCMC method of sampling.

The estimations of model parameters presented in Table 1 are obtained from the generated posterior samples.

The maximum likelihood estimate (MLE) method is introduced to compare the Bayesian method. The estimations of model parameters with MLEs are shown in Table 2, and there are close to the estimations of model parameters with Bayesian method.

As discussed above, when the failure threshold is set as 300, the reliability of the CNC machine tools can be obtained using Eq. (7), as presented in Fig. 2. From Fig. 2, it can be concluded that the reliability curves obtained by using the parameters obtained from the two methods are similar. However, the MLE method cannot consider the prior information, which limits its subsequent applications.

The reliability analysis results are inaccurate compared to the results obtained by the project [28]. Lack of effective prior information is a major cause of this problem.

Table 2. Estimations of model parameters with MLE and the 95 % confidence intervals.

	Estimation	CI.lower	CI.upper
η	0.2315	0.1795	0.2894
δ	13.28	2.112	34.63
γ	12.22	1.596	32.92

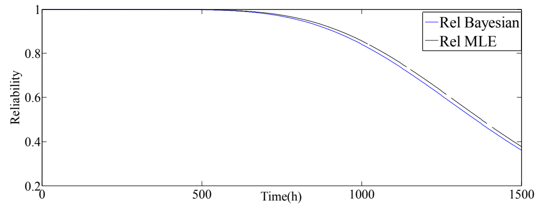

Fig. 2. Reliability of the CNC machine tools.

New prior distributions of the model parameters are quantified from the subjective information of experts and the historical data:

$$\delta \sim Ga(16,1), \gamma \sim Ga(16,64), \eta \sim U(0,10)$$

where the prior distribution of the parameters δ and γ are quantified from historical data [28], the prior distribution of parameter η is obtained by the interval method in Ref. [20]. The parameter estimation results obtained from the population and each sample's degradation observations are presented in Table 3.

The trace plots of model parameters are shown in Fig. 3, it can be noted that all values are within a zone without strong periodicities and tendencies. This indicates when using openBUGS to obtain posterior samples of the parameters, the iteration is convergent.

In this paper, the fitted lines are compared to the estimation results with the degradation observations. Applying the performance evolution prediction method provided by Eq. (11), the performance evolution curve of each sample based on the parameter estimation results can be obtained from each sample's degradation observations. It could show the modeling ability of the proposed model and the prediction ability of the proposed method. The comparison results are shown in Fig. 4.

When the failure threshold is set as 300, the reliability of the CNC machine tools can be obtained by using Eq. (7) and is presented in Fig. 5. From Fig. 5, it is obvious that the reliability curves of different samples for different CNC machine tools are different. These differences in the reliability curves demonstrate the necessity of introducing the random effect into the degradation model for reliability analysis of the CNC machine tools.

The reliability assessment of the CNC machine tools is consistent with the engineering experience [28]. This indicates that the proposed model and parameter estimation method are suitable for the CNC machine tools degradation analysis.

Table 3. Estimations of model parameters with informative prior distribution.

	Mean	Standard deviation	Confidence interval interval	
			2.5 %	97.5 %
$\eta_{\text{population}}$	0.2675	0.03536	0.2043	0.3418
$\delta_{\text{population}}$	9.445	2.221	5.667	14.35
$\gamma_{\text{population}}$	7.392	1.946	4.31	11.89
$\eta_{\text{sample 1}}$	0.4238	0.09812	0.2602	0.643
$\delta_{\text{sample 1}}$	12.48	3.047	7.34	19.08
$\gamma_{\text{sample 1}}$	5.286	1.484	3.092	8.794
$\eta_{\text{sample 2}}$	0.2239	0.05217	0.1344	0.3359
$\delta_{\text{sample 2}}$	11.73	2.9	6.826	18.06
$\gamma_{\text{sample 2}}$	5.483	1.593	3.168	9.331
$\eta_{\text{sample 3}}$	0.2238	0.05206	0.1348	0.3392
$\delta_{\text{sample 3}}$	12.54	3.045	7.367	19.23
$\gamma_{\text{sample 3}}$	5.262	1.47	3.063	8.802
$\eta_{\text{sample 4}}$	0.5695	0.1446	0.3277	0.8924
$\delta_{\text{sample 4}}$	14.01	3.361	8.29	21.34
$\gamma_{\text{sample 4}}$	4.868	1.295	2.928	7.914
$\eta_{\text{sample 5}}$	0.5629	0.1339	0.3304	0.8461
$\delta_{\text{sample 5}}$	13.23	3.191	7.826	20.23
$\gamma_{\text{sample 5}}$	5.068	1.385	2.992	8.42

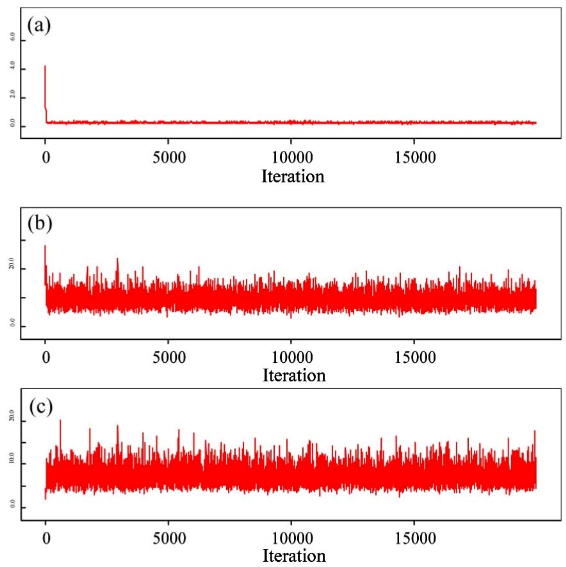

Fig. 3. The trace plots of model parameters: (a) η ; (b) δ ; (c) γ .

5. Conclusions and future work

In this paper, a degradation model considering the non-homogeneity of units is presented, and the reliability of CNC machine tools is characterized with gamma process. The performance degradation analysis and reliability assessment method is proposed upon Bayesian method, which includes

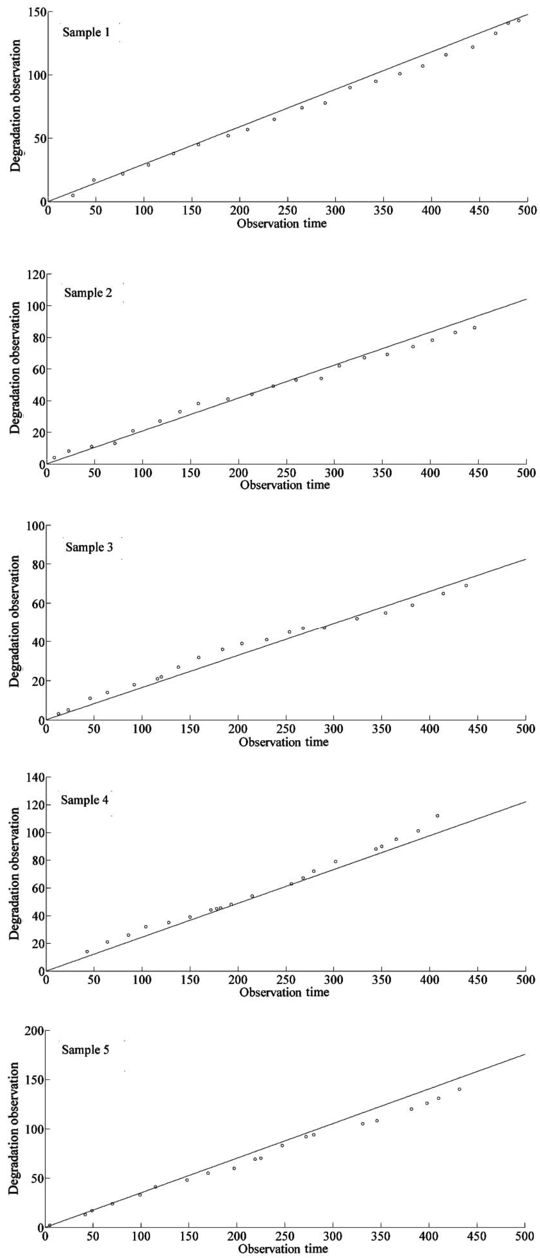

Fig. 4. The comparisons of fitted lines with the degradation observations.

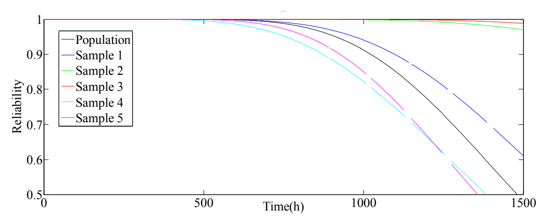

Fig. 5. Reliability of the CNC machine tools with informative prior distribution.

parameter estimation based on MCMC method and prediction of performance degradation considering unit non-homogeneity. This model is suitable for the reliability assessment of CNC machine tools under non-informative or informative prior of MCMC method. To illustrate this approach, the method is applied to a CNC machine tools to predict the degradation of the position accuracy of tools.

Additional practical issues can be considered in future studies. First, since CNC machine tools are complex electromechanical systems, the multiple degradation processes could be applied for the reliability assessment. Second, to make the model more in line with the actual engineering, the operation conditions and working stresses of CNC machine tools could be factored into the degradation model.

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Nomenclature

PDF : Probability density function
CDF : Cumulative distribution function
$X(t)$: Independent incremental process
$\eta(t)$: Shape parameter function of the gamma process
$g(x)$: The probability density function of gamma process
$Ga(\eta\Delta t,\ v^{-1})$: Gamma process
v^{-1} : Scale parameter of gamma process
C : The failure threshold
$R(t)$: Reliability function of the product
$F_{2\eta n,2\delta}(x)$: CDF of F-distribution
$\pi(\eta)$: Prior distribution of model parameter η
$\pi(\gamma^{-1})$: Prior distribution of model parameter γ^{-1}
$\pi(\delta)$: Prior distribution of model parameter δ
D : The degradation observations
$L(D\,|\,\eta,\gamma^{-1},\delta)$: Likelihood function of degradation observations D
$p(\eta,\gamma^{-1},\delta|D)$: Posterior distribution of model parameters

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