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Total fatigue life prediction for welded joints based on initial and equivalent crack size determination

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Abstract

When the linear elastic fracture mechanics-based approaches are used to predict the fatigue life of welded joints, initial crack size is a key point, which eventually affects the accuracy of total fatigue life prediction. Meanwhile, the life prediction process under random loading is complicated. In this paper, a novel method is proposed to determine the initial crack size, which is based on the results of back-extrapolation approach. The proposed method expresses the stress intensity factor, and the boundary between crack initiation and propagation period is taken into consideration. Based on the proposed method, deterministic total fatigue life can be obtained with fewer tests and less cost. In addition, the concept of equivalent crack size and its calculation model are proposed to reduce the complexity of the calculation process of fatigue life prediction under random loading, and model uncertainty is included into the prediction model of probabilistic fatigue life based on equivalent crack size. It is feasible, which has been verified, to take the influence of stress level into account when determining the initial crack size. Meanwhile, the proposal of equivalent crack size simplifies the calculation process of probabilistic fatigue life, and the consideration of model uncertainty is more conducive to assess the safety and reliability of the materials or structures.

Keywords

Total fatigue life prediction, initiation life, initial crack size, equivalent crack size, welded joints

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Introduction

Welding is a widely used manufacturing method due to its advantages of high strength, flexibility, good sealing ability, and simple preparation of the welded structures (Barsoum, 2008). However, welded structures also have their disadvantages of performance nonuniformity, stress concentration, and residual stress for different defects such as undercut, nonfusion, porosity, and cracks in welded joints (Yan, 2011). Fatigue fracture is the main failure mode of welded structures. Under service loading, crack propagation and even unstable fracture can be caused by welded joints. The employment of advanced reliability analysis technologies, such as reliability assessment and fatigue life prediction are important for the improvement of service life for mechanical structures (Peng et al., 2016, 2017; Al-Mukhtar et al., 2010; Li et al., 2015, 2016; Zuo et al., 2015; Liu and Mahadevan, 2009; Peng et al., 2016). Welded joints are usually the weakest link in welded structures; therefore, an appropriate fatigue life prediction method is required for welded structures.

In engineering, two approaches are widely used to predict fatigue life of mechanical structures: the *S–N* curve of materials combined with the damage accumulation theory-based approach; the fracture mechanics and crack growth curve-based approach (Al-Mukhtar et al., 2010; Berto and Ayatollahi, 2011; Gao et al., 2015; Lazzarin et al., 2014; Liu and Mahadevan, 2009; Lv et al., 2015; Pook et al., 2014). The second approach can be classified into two categories: linear elastic fracture mechanics (LEFM) and elastic–plastic fracture mechanics (EPFM). For welded structures, the EPFM-based method is recommended to predict fatigue life. However, when the size of plastic zone is small, LEFM can be applied to predict the fatigue life (Carpinteri et al., 2015; Harrison, 1969). When applying the LEFM-based approach, the determination of initial crack size is one of the critical problems which will affect the accuracy of total fatigue life prediction.

Total fatigue life is often considered as propagation life or the sum of initiation and propagation life. It should be noted that the viewpoint that the total fatigue life is equal to crack propagation life has been accepted by some researchers based on generally existing welding defects in welded joints (Mikkola et al., 2014; Shen and Choo, 2012; Taghizadeh et al., 2013). But others point out that the initial crack phase takes up a certain proportion of total fatigue life for welded structures (Lassen, 1990; Wang and Jones, 2007). When crack initiation life is considered as a part of total fatigue life, it is often treated as the number of cycles when cracks reach a specified size, that is initial crack size a_{p0}, such as 0.1 mm (Lassen, 1990), 0.15 mm (Zhang and Maddox, 2009), 0.25 mm (Lawrence et al., 1978), 0.36 mm (Zhang and Maddox, 2009), and 0.5 mm (ABS, 2003). Thus, it is reasonable to hold the opinion that crack initiation life depends on initial crack size. Lassen (1990) found that if initial crack size was taken as 0.1 mm, 30–40% of total fatigue life of welded joints was spent in crack initiation phase. If it is taken as 0.25 or 0.5 mm, crack initiation life would have a greater percentage. Through taking a specified size as a_{p0}, initiation life can be described, but human factors and uncertainty are not considered. Large differences between initiation life predictions obtained by different initial crack sizes will come into being, and the orders of magnitude difference in total fatigue life prediction may occur eventually.

The determination of initial crack size a_{p0} plays an important role in the analysis of fatigue crack propagation. For crack propagation life prediction, the initial crack size a_{p0} is taken as the value in the scope of 0.0151–0.5 mm (Zhang, 2013). For welded structures, the similar method is used to determine initial crack size a_{p0}. Al-Mukhtar et al. (2010) investigated the cracks initiated from the weld toe and the weld root, they determined the weld toe crack to be equal to 0.1 mm, whereas the weld root crack size was varied depending on the degree of the weld penetration. Okawa et al. (2013) assumed that initial cracks were semicircular with value 0.15 mm. The other values, such as 0.0151, 0.5, and 1 mm are also assumed to be initial crack size (ABS, 2003; Krasovskyy and Virta, 2014; Zhang, 2013). An alternative approach for initial crack size determination is adopted based on nondestructive inspection (NDI) techniques (Fazeli and Mirzaei, 2012; Kim et al., 2012). But non-conservative predictions will be obtained if initial crack size is smaller than the current detection precision. In addition, effective initial flaw size (EIFS) is defined and utilized to replace the initial crack size (Liu and Mahadevan, 2009). Liu and Mahadevan (2009), Lu et al. (2010), and Xiang et al. (2010) took EIFS as a material property. However, it can be found that EIFS was related to stress level (Molent et al., 2006; White et al., 2005). Thus, there was a difference between the two methods, and it is still a problem to determine initial crack size for crack propagation life prediction.

As mentioned, when the LEFM approach is used to predict fatigue life, the current investigation focused on the processing of initial crack size. Meanwhile, it should be noted that the calculation of fatigue life under constant amplitude fatigue loading is slightly simpler, but complicated calculation will be faced under random loading conditions. Thus, when the method based on fracture mechanics theory combined with a crack growth rate curve is used to predict the fatigue life of welded joints, the initial crack size is difficult to determine, and the life prediction process under random loading is complicated. For the first problem, although many works have been done searching for an appropriate method for initial crack size determination, there are some controversies and drawbacks for different methods. Hence, an exploratory study is carried out and a new method for initial crack size determination considering loading conditions is proposed in this paper firstly. And then, a practical procedure is proposed for deterministic total fatigue life prediction. Thirdly, according to the procedure, equivalent crack size is defined as an exponential function of load amplitude to conveniently predict probabilistic total fatigue life under random loading. The feasibility and accuracy of the proposed method are demonstrated by the comparison between the predictions and experimental data for welded joints.

Deterministic total fatigue life prediction for welded joints based on a new approach for initial crack size determination

As mentioned above, for the common methods for total fatigue life prediction, initial crack size determination is a key point in the whole process, which will affect the accuracy of total fatigue life prediction. A practical method for initial crack size determination is proposed in this section. The derivation of the new method will be presented in detail after introducing the model for crack initiation and propagation life prediction.

Crack initiation life prediction

In this section, a crack initiation life model for welded joints which is based on the method proposed by Zheng (1993) is derived. The derivation of the crack initiation life prediction model can be described as follows. First, according to local stress–strain approach, fatigue life for smooth specimen can be expressed as

$$N_f = \frac{\varepsilon_f^2}{\left(\Delta\varepsilon - \Delta\varepsilon_c^2\right)^2} \tag{1}$$

where $\Delta\varepsilon$ is cyclic strain range, $\Delta\varepsilon_c$ is the theoretical strain fatigue limit, and ε_f is fracture ductility.

For high strength steel, when strain hardening exponent is relatively small, local strain at the root of notch $\Delta\varepsilon$ is approximately equal to

$$\Delta\varepsilon = 2\left(\frac{\Delta\sigma_{eqv}^2}{EK}\right)^{\frac{1}{1+n}} \tag{2}$$

where E is the elastic modulus; n and K are strain hardening exponent and strength coefficient, respectively; and $\Delta\sigma_{eqv}$ is equivalent load amplitude and it can be calculated by

$$\Delta\sigma_{eqv} = K_t \Delta\sigma \sqrt{\frac{1}{2(1-R)}} \tag{3}$$

where K_t is the stress concentration factor, $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$ is the load amplitude, and R indicates the stress ratio, namely $R = \sigma_{\min}/\sigma_{\max}$.

Based on the formula that $\sigma_f = K\left(\varepsilon_f\right)^n$, the expression of fatigue crack initiation life N_i for notch specimen is obtained by substituting equation (2) into equation (1), and replacing N_f in equation (1) with N_i, yields

$$N_i = \frac{1}{\left[\frac{2}{\varepsilon_f}\left(\frac{\Delta\sigma_{eqv}^2}{EK}\right)^{\frac{1}{1+n}} - \frac{\Delta\varepsilon_c}{\varepsilon_f}\right]^2} = \frac{0.25\left(\sqrt{E\sigma_f\varepsilon_f}\right)^{\frac{4}{1+n}}}{\left[\Delta\sigma_{eqv}^{\frac{2}{1+n}} - \frac{\Delta\varepsilon_c}{2\varepsilon_f}\left(E\sigma_f\varepsilon_f\right)^{\frac{1}{1+n}}\right]^2} \tag{4}$$

Suppose that

$$B = 0.25\left(\sqrt{E\sigma_f\varepsilon_f}\right)^{\frac{4}{1+n}} \tag{5}$$

$$\left(\Delta\sigma_{eqv}\right)_{th} = \sqrt{E\sigma_f\varepsilon_f}\left(\frac{\Delta\varepsilon_c}{2\varepsilon_f}\right)^{\frac{1+n}{2}} \tag{6}$$

The expression of crack initiation life is rewritten as

$$N_i = \frac{B}{\left[\Delta\sigma_{eqv}^{\frac{2}{1+n}} - \left(\Delta\sigma_{eqv}\right)_{th}^{\frac{2}{1+n}}\right]^2} \tag{7}$$

Equations (5) to (7) are the basic formulas of fatigue crack initiation life, which reveal the relation among fatigue initiation life and cyclic loading condition ($\Delta\sigma$, R), geometry property K_t, and fatigue crack initial threshold $\left(\Delta\sigma_{eqv}\right)_{th}$. Zheng (1993) pointed out that the calculation of coefficient B was related to the crack initiation mechanism and depended on the microstructure. For the alloy with duplex microstructure, the value of B can be obtained by equation (5), and for the alloy with homogeneous microstructure, B is calculated by the following expression

$$B = 0.25(0.1E)^{\frac{4}{1+n}} \tag{8}$$

By analyzing the experimental data and predicted results, it is concluded that the crack initial threshold $\left(\Delta\sigma_{eqv}\right)_{th}$ using equation (6) is conservative, and the method to determine $\left(\Delta\sigma_{eqv}\right)_{th}$ needs further research. Based on this, Lü et al. (1993) concluded that $\left(\Delta\sigma_{eqv}\right)_{th}$ is equal to fatigue limit σ_{-1} through detailed derivation with the verified accuracy, that is

$$\left(\Delta\sigma_{eqv}\right)_{th} = \sigma_{-1} \tag{9}$$

In this way, crack initiation life of notched specimens is obtained by combining equations (3) and (7) to (9). A welded joint often contains stress concentration because of welding defects, therefore it can be considered as notched specimen, that is, this method can be used for crack initiation life prediction for welded joints (White et al., 2005). Then, crack initiation life of welded joints can be obtained by substituting equations (3), (8), and (9) into equation (9)

$$N_i = \frac{0.25(0.1E)^{\frac{4}{1+n}}}{\left[\left(K_t\Delta\sigma\sqrt{\frac{1}{2(1-R)}}\right)^{\frac{2}{1+n}} - \sigma_{-1}^{\frac{2}{1+n}}\right]^2} \tag{10}$$

Crack propagation life prediction

Currently, crack propagation life is predicted on the basis of crack growth rate function. Until now, a large number of researchers have devoted themselves to investigating the models and theories on the development of crack growth. Compared with other models, Paris law has no consideration of the effects of stress ratio, fracture toughness, crack growth threshold, and crack closure, but it has simple form and does not introduce parameters which are difficult to determine, therefore Paris law is widely used in engineering (Ni et al., 2006; Paris and Erdogan, 1963). According to the Paris law, crack growth rate is expressed as

$$\frac{da}{dN} = C(\Delta K)^m \tag{11}$$

When material parameters C and m are determined, crack propagation life is obtained by integrating the following expression

$$N_p = \int_{a_{p0}}^{a_f} \frac{da}{C(\Delta K)^m} \tag{12}$$

where a is the crack size, N_p is the fatigue propagation life, and C and m are material parameters for a given loading and environment condition. ΔK is the stress intensity factor at crack tip, a_{p0} and a_f indicate initial size and finial size when fracture occurs, respectively.

Crack propagation life of welded joints can be predicted by using Paris law; the procedure can be summarized as follows

(1) Determining the relevant parameters of the growth rate curve associated with material properties, types of welded joints, and standard of fatigue life prediction.

(2) Calculating the stress intensity factor ΔK.
(3) Determining the initial crack size a_{p0} and finial crack size a_f.
(4) Integrating the propagation rate function to obtain the fatigue life of welded joints.

Among the above steps, step 3) is a critical one. The finial crack size a_f does not much affect the accuracy of propagation life prediction, which has been accepted in many research works (Al-Mukhtar et al., 2010). One practice is to determine a_f according to fracture toughness and stress levels (Liu and Mahadevan, 2009). And the alternative method is to take an appropriate value according to the thickness of specimen, such as one-third, one-half, and the whole of thickness (Al-Mukhtar et al., 2010; Lassen, 1990; Shen and Choo, 2012). Compared with a_f, initial crack size a_{p0} has a great influence on propagation life prediction, a little discrepancy of a_{p0} sometimes can give rise to much difference in predicted life (Al-Mukhtar et al., 2010). Thus, the method to determine initial crack size a_{p0} which can yield accurate propagation life prediction is needed.

Determination of initial crack size

The main problem in predicting propagation life using LEFM theory is to determine the initial crack size. So far, there are two main methods to realize the determination of the initial crack size: NDI method and empirical method. Because the NDI techniques cannot detect the initial crack size which is below the current detection precision, a dangerous fatigue life prediction will be produced if the initial crack size is taken as the NDI detection limit (Liu and Mahadevan, 2009). For the alternative way, initial crack size is often taken the value in the range of a fraction of a millimeter to a few millimeters, thus the accuracy cannot be guaranteed and uncertainties will be introduced through this method, a relatively little difference in initial crack size sometimes will give large difference in propagation life prediction (Duan et al., 2000). In addition, equivalent initial flaw size (EIFS) was proposed for determining the initial crack size (Liu and Mahadevan, 2009). Based on the formula of fatigue limit σ_{-1} expressed by fatigue crack growth threshold ΔK_{th} and a fictitious crack size a, as shown in equation (13), Liu and Mahadevan (2009) used an approach to obtain EIFS and its distribution

$$\Delta K_{th} = \sigma_{-1} Y \sqrt{\pi a} \tag{13}$$

where Y is geometry correction factor.

In this way, initial crack size is independent of loading conditions. But back-extrapolation method was used by Molent et al. (2006) and White et al. (2005) to derive EIFS, and the results were in good agreement with experimental data, thus it can be said that initial crack size was related with stress level. Therefore, there are some drawbacks or controversies in the common methods for initial crack size determination, and further research is needed.

In this paper, a new method for initial crack size determination is proposed with the adequate consideration of the controversy in EIFS determination methods, rather than using some fictitious values. First, the relation between initial crack size and stress level is considered based on the results of back-extrapolation method. Lazzarin et al. (2008a, 2008b, 2009, 2010) have done a lot of research work and proposed a strain energy density method which can calculate notch stress intensity factors rapidly. Herein, according to the model in

BS7910 (2005), the stress intensity factor ΔK has connection with loading amplitude and crack size, that is

$$\Delta K = Y\Delta\sigma\sqrt{\pi a} \tag{14}$$

Meanwhile, combining the consideration that the crack initial period is regarded as the situation when $\Delta K < \Delta K_{th}$ is satisfied, and crack starts to propagate only when $\Delta K > \Delta K_{th}$, the initial crack size is assumed to be a function of fatigue crack growth threshold ΔK_{th} and stress level $\Delta\sigma$, as shown in equation (15)

$$a_{p0} = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma Y}\right)^2 \tag{15}$$

It should be noted that fatigue crack growth threshold ΔK_{th} is defined as the stress intensity factor when crack growth rate tends to zero. However, it is difficult to measure. In engineering, its value is accepted when crack growth rate is equal to 10^{-7} mm/cycle (Wang, 2002) since the crack increment cannot be precisely measured by the existing test methods if $\mathrm{d}a/\mathrm{d}N < 10^{-8}$. In addition, the calculation of geometry correction factor Y is affected by crack configuration. In general, cracks are assumed as semielliptical shape in propagation phase, and the crack aspect ratio $a/2c$ is generally used to represent crack shape. Lassen (1990) carried out a linear regression analysis of 75 semielliptical cracks, and found that the relationship between crack depth a and half crack length c accorded with equation (16). Combining welded details with the result of linear regression analysis, geometry correction factor Y for welded joints with semielliptical shape cracks can be derived subsequently (Lassen, 1990)

$$2c = 2.92a + 3.83 \tag{16}$$

Procedure of total fatigue life prediction based on initial crack size determination

According to the above research and the challenges existing in total fatigue life prediction for welded joints structures, total fatigue life can be predicted by the following steps:

(1) The stress concentration factor K_t; fatigue limit σ_{-1}; strain hardening n; and crack propagation parameters C, m; and the expression of stress intensity factor ΔK are determined according to the type of material and the form of welded joint.
(2) The parameter B, equivalent load amplitude $\Delta\sigma_{eqv}$, and crack initial size a_{p0} are calculated by using equations (2) and (3) and equation (10).
(3) Crack initiation life and propagation life are derived according to the above analysis and calculation. And total fatigue life is obtained by adding these two factors together.

This procedure is based on the proposed method for initial crack size determination. The crack initiation life is independent of initial crack size, and when propagation life is calculated, initial crack size is not taken as a fictitious value, thus, there is less uncertainty comparing with the existing methods. Meanwhile, the consideration of the relationship between initial crack size and loading conditions conforms with the results of the back-extrapolation method. Another important point is that geometry correction factor Y probably is not a constant, but a function of crack size, thus initial crack size is the solution of equation (14) when $\Delta K = \Delta K_{th}$.

Model validation

The accuracy and feasibility of the proposed method is discussed by comparing experimental data with the predicted total fatigue life. Total fatigue life will be predicted by the proposed method and the common treatments, that is taking propagation life with a fictitious initial crack size as total fatigue life, and summing experimental crack initiation life and predicted propagation life with a specified initial crack size. Experimental data are collected from welded joints made from 16Mn low alloy steel, its mechanical properties are shown in Table 1, and the geometry of welded joints can be seen in Figure 1. Stress relief annealing was conducted before welding. And the CO_2 shielded welding and H08Mn2SiA wires with a diameter of 1.6 mm were used in welding. The welding specifications are listed in Table 2.

Table 1. Mechanical properties of 16Mn steel (Lawrence et al., 1978; Zhu et al., 2008).

Yield limit (MPa)	348
Tensile strength (MPa)	532
Young's modulus (GPa)	201
Fatigue limit (MPa)	280.8
Fracture stress (MPa)	758
Strain hardening exponent	0.13
Reduction of cross section	65%
Ductility	25%

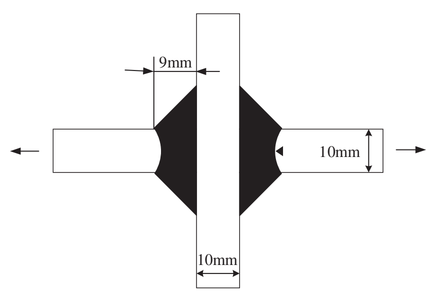

Figure 1. Geometry of welded joints.

Table 2. Welding specifications of 16Mn steel welded joints (Lawrence et al., 1978).

Welding current (A)	348
Arc voltage (V)	532
Welding speed (mm/min)	201
Gas flow rate (l/min)	289.2

All tests were carried out on a fatigue test machine at room temperature, and constant amplitude sinusoidal wave load was applied under the loading frequency 50 Hz. The detailed experimental process and mechanical properties can be found in Lawrence et al. (1978). And the maximal $\sigma_{\max}$ and minimal loading $\sigma_{\min}$, load amplitude $\Delta\sigma$, stress ratio R, and experimental fatigue life are listed in Table 3.

According to the proposed procedure, initiation and propagation life can be predicted separately, and total fatigue life is obtained by adding them together. Crack initiation life for 16Mn steel welded joints is first predicted in this section. When using crack initiation life prediction model in equation (10), and the material properties listed in Table 1 and experimental data, crack initiation life for 16Mn steel welded joints is given by

$$(N_i)_{\text{welded joints}} = \frac{0.25\left(0.1 \times \left(2.01 \times 10^5\right)\right)^{3.5398}}{\left[\left(K_t \Delta\sigma \sqrt{\frac{1}{2(1-R)}}\right)^{1.7699} - (289.2)^{1.7699}\right]^2} \tag{17}$$

According to Atzori–Lazzarin's diagram (Atzori and Lazzarin, 2001), the high-cycle fatigue limits for blunt cracks are determined either by means of ΔK_{th} or the theoretical stress concentration factor K_t. In Lawrence et al. (1978), K_t was taken as 4.0, thus the same value is also taken here to

Table 3. Fatigue life tested results of 16Mn steel welded joints (N_i—crack initiation life when a_{p0} is taken 0.25 mm; N_f—total fatigue life) (Lawrence et al., 1978).

Sequence	$\sigma_{\max}$ (MPa)	$\sigma_{\min}$ (MPa)	R	$\Delta\sigma$ (MPa)	N_i (cycles)	N_f (cycles)
1	250	25	0.1	225	320,000	507,000
2	250	25	0.1	225	97,000	175,000
3	250	25	0.1	225	279,000	445,000
4	250	25	0.1	225	120,000	290,000
5	250	25	0.1	225	201,000	314,000
6	250	75	0.3	175	370,000	501,000
7	250	75	0.3	175	240,000	350,000
8	250	75	0.3	175	174,000	247,000
9	250	75	0.3	175	470,000	610,000
10	225	45	0.2	180	450,000	750,000
11	225	45	0.2	180	280,000	454,000
12	225	45	0.2	180	410,000	592,000
13	225	45	0.2	180	310,000	540,000
14	225	45	0.2	180	415,000	678,000
15	200	60	0.3	140	750,000	1,103,000
16	200	60	0.3	140	386,000	697,000
17	200	60	0.3	140	1,004,000	>2,000,000
18	200	60	0.3	140	602,000	970,000
19	200	60	0.3	140	720,000	1,036,000

predict initiation life. Substituting load amplitude $\Delta\sigma$ and stress ratio R, crack initiation life predictions can be obtained and the results can be seen in Table 4.

Next, propagation life is calculated based on the proposed method to determine initial crack size a_{p0}. First, based on the statistical results in Zhang (2013), the mean values of material parameters C and m are taken for propagation life calculation, namely $C = 1.85 \times 10^{-13}$ and $m = 3.0$. Then, according to the above-mentioned discussion, fatigue crack growth threshold ΔK_{th} is obtained when crack growth rate is equal to 10^{-7} mm/cycle. In this way, initial crack sizes under different loading conditions are obtained by substituting ΔK_{th} and load amplitude $\Delta\sigma$ into equation (16). Finally, by substituting initial crack size a_{p0}, material parameters C and m into Paris law, crack increment history is obtained. The evolution results under the loading condition that load amplitude $\Delta\sigma$ is equal to 140 MPa, is drawn in Figure 2. Crack increment curves when three fictitious values,

Table 4. Crack initiation life prediction of 16Mn steel welded joints.

Load amplitude (MPa)	Crack initiation life (cycles)
225	72,800
175	127,470
180	154,590
140	428,020

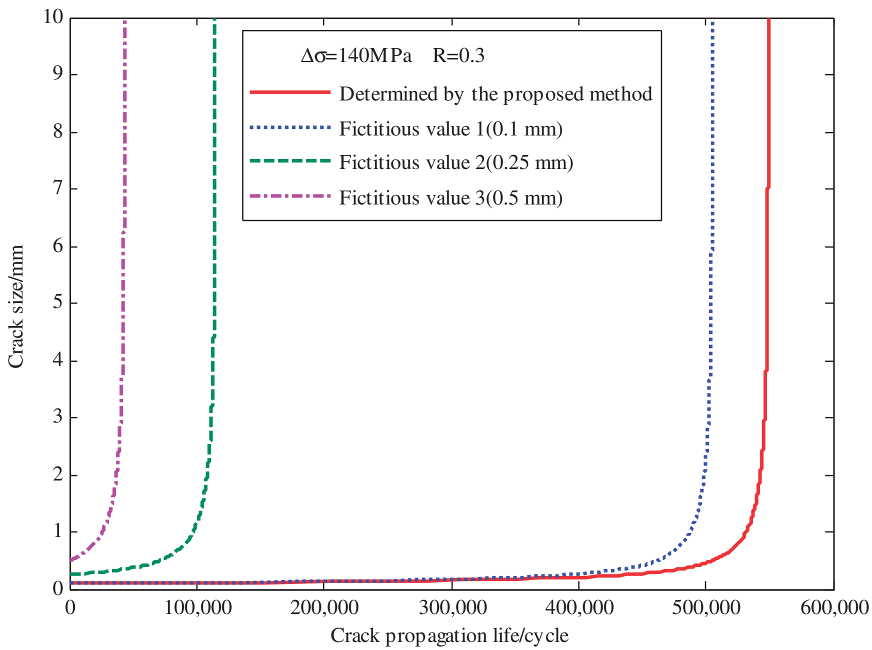

Figure 2. Crack increment history at $\Delta\sigma = 140$ MPa with initial crack size using fictitious values and calculating by the proposed method in "Deterministic total fatigue life prediction for welded joints based on a new approach for initial crack size determination" section.

namely 0.1, 0.25, and 0.5 mm, are taken as initial crack size, as shown in Figure 2. According to Figure 2, under the same loading condition, even the difference of initial crack size is just a fraction of a millimeter, crack increment history is largely different, such as the three fictitious values. In view of the experimental data listed in Table 3, if the total fatigue life is considered as the distribution in propagation phase, only taking one of the three fictitious values, namely 0.1 mm, can give the evolution result which is relatively close to the real fatigue life. Thus, it is difficult to choose an appropriate a_{p0} to guarantee the prediction accuracy if total fatigue life is regarded as the cycles spending in propagation period. However, the prediction produced by the proposed method is a certain curve for a given loading condition and welded joint and is close to the experimental data. The same phenomenon can also be found under the other loading conditions, that is load amplitude $\Delta\sigma$ is equal to 225, 175, and 180 MPa.

According to the procedure in "Procedure of total fatigue life prediction based on initial crack size determination" section, which is based on the proposed method for initial crack size determination, propagation life of 16Mn steel welded joints can be obtained when finial crack size a_f is determined. As previously mentioned, life prediction accuracy is tightly related to initial crack size a_{p0} and relatively unaffected by a_f (Gao et al., 2015). In recent investigations, the determination of a_f is in connection with the thickness of specimen. One-third, one-half, and the whole of thickness are often taken as the finial crack size (Gao et al., 2015; Lassen, 1990; Zhang, 2013), and one-third of thickness is taken in this paper to predict propagation life. Thus, deterministic total fatigue life of 16Mn steel welded joints can be calculated according to the proposed procedure, through summing initiation life and propagation life. The predicted and experimental total fatigue lives under the four loading conditions are shown in Figure 3, In Figure 3, the icons "*" and "o" represent the experimental data and the average experimental data under different loading conditions. And the icons "△" are the total fatigue lives predicted by summing initiation and propagation life, and initiation life is taken as the cycles when crack size reaches to 0.25 mm, and propagation life is obtained by integrating equation (11) with the specified initial crack size. The red icons "+" represent the predicted results based on the proposed method. It can be found that although the method, marked by (1), can give an acceptable prediction accuracy, the specified initiation life when $a_{p0} = 0.25$ mm should be obtained by tests, thus more time and cost are needed. However, the life predictions calculated by the proposed method, marked by (2), are close to the average tested results of total fatigue life, and less requirement for tests.

Probabilistic total fatigue life prediction under random loading based on equivalent crack size determination

Equivalent crack size and its distribution determination

According to the results in "Deterministic total fatigue life prediction for welded joints based on a new approach for initial crack size determination" section, the procedure based on the proposed method for initial crack size determination can be used to predict total fatigue life for welded joints with a good approximation of the mean value of experimental life. The expression of total fatigue life is

$$N_{total} = N_i + N_p = \frac{B}{\left[\left(K_t \Delta\sigma\sqrt{\frac{1}{2(1-R)}}\right)^{\frac{2}{1+n}} - (\Delta\sigma_{-1})^{\frac{2}{1+n}}\right]^2} + \int_{a_{p0}}^{a_f} \frac{\mathrm{d}a}{C\left(Y\Delta\sigma\sqrt{\pi a}\right)^m} \tag{18}$$

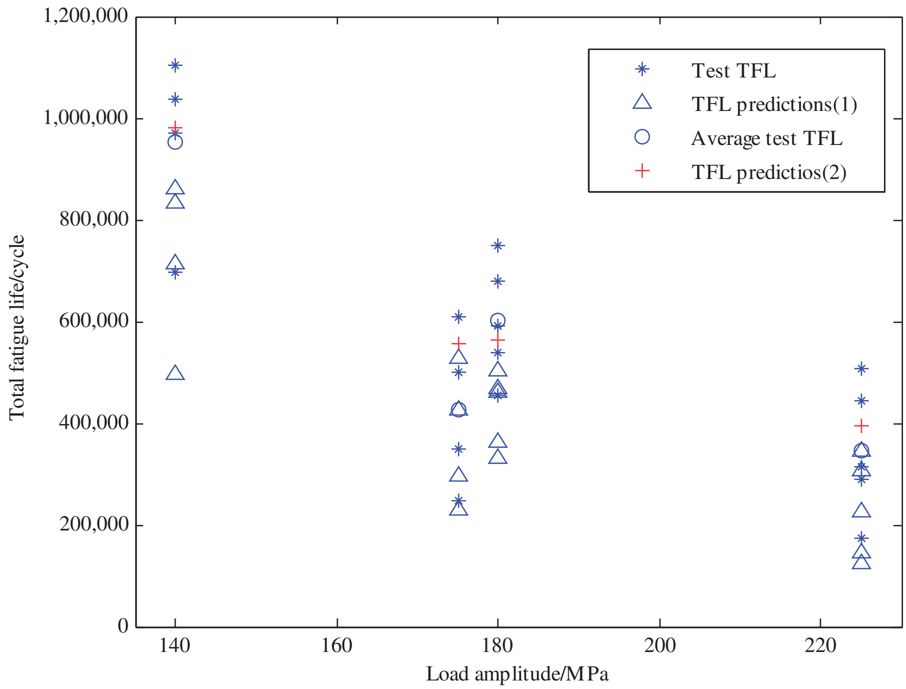

Figure 3. Comparison results of experimental and predicted total fatigue life prediction ((1) TFL predicted by summing experimental initiation life and predicted propagation life with specific $a_{p0} = 0.25$ mm and (2) TFL predicted by the proposed method for a_{p0} determination).
TFL: total fatigue life.

Deterministic total fatigue life can be conveniently calculated when constant amplitude fatigue loading is applied. However, structures are usually subjected to variable amplitude or random loading in engineering (Chen et al., 2013; Gates and Fatemi, 2015; Yan et al., 2000). Although total fatigue life can be obtained by traditional cycle-by-cycle calculation and integration techniques, the computational procedure is complex because the complexity of the geometry correction factor Y and its substitution is needed to apply repeatedly. Therefore, a new method is proposed for solving this problem, which is based on the determination of equivalent crack size and its distribution.

In equation (18), total fatigue life consists of two parts: crack initiation life and crack propagation life. According to integral characteristics, the number of cycles spent in crack initial phase can be regarded as a part of propagation life. Then, an equivalent crack size a_{eqv} can be found and crack initiation life can be equivalent to the cyclic number when crack grows to a_{p0}. In this way, total fatigue life can be rewritten as

$$N_{total} = N_i + N_p = \int_{a_{eqv}}^{a_f} \frac{\mathrm{d}a}{C(\Delta K)^m} \tag{19}$$

Thus, the assumption that crack initial phase is equivalent to the lower portion of propagation phase, when the crack is assumed to propagate from an equivalent size to the initial crack size a_{p0}, is considered in this paper. Along with the research work presented in Molent et al. (2008), the assumption that the equivalent crack size a_{eqv} has relationship with load amplitude, and the equivalent crack size a_{eqv} is exponential to load amplitude, is reasonable. That is

$$a_{eqv} = k(\Delta\sigma)^b \tag{20}$$

where a_{eqv} is the equivalent crack size, $\Delta\sigma$ is the load amplitude, and k and b are material constants which can be obtained through experiment.

Therefore, the equivalent crack size can be derived just when the load condition is known. Under constant amplitude loading, a_{eqv} can be directly obtained by using equation (20), and the total fatigue life is the number of cycles when crack propagates from the equivalent size to the critical size a_f. In the case of random loading, the form of the probability distribution of the equivalent crack size can be derived according to the above expression, where the probability density function of fatigue loading is a random variable.

According to equation (14) and the above discussion, the total fatigue life of welded joints is

$$N_{total} = \int_{a_{eqv}}^{a_f} \frac{\mathrm{d}a}{C(\Delta K)^m} = \int_{a_{eqv}}^{a_f} \frac{\mathrm{d}a}{C\left(Y\Delta\sigma\sqrt{\pi a}\right)^m} = \frac{1}{C\left(\Delta\sigma\sqrt{\pi}\right)^m}\int_{a_{eqv}}^{a_f} \frac{\mathrm{d}a}{\left(Y\sqrt{a}\right)^m} \tag{21}$$

Obviously, the expression $\int_{a_{eqv}}^{a_f} \frac{\mathrm{d}a}{\left(Y\sqrt{a}\right)^m}$ is a function of a_{eqv}, thus equation (22) can be used to describe the total fatigue life if set that $f(a_{eqv}) = \int_{a_{eqv}}^{a_f} \frac{\mathrm{d}a}{\left(Y\sqrt{a}\right)^m}$

$$N_{total} = \frac{1}{C\left(\Delta\sigma\sqrt{\pi}\right)^m} f\left(a_{eqv}\right) \tag{22}$$

In this way, total fatigue life can be predicted for both constant amplitude and random loading through a defined equivalent crack size. By substituting load amplitude into equation (22), then total fatigue life can be obtained through integration under constant amplitude loading. And for random loading, by using probability density function of fatigue loading, probabilistic fatigue life prediction is also obtained through equation (22). In this method, the geometry correction factor Y is calculated for just one time, which simplifies the complexity of calculation.

Next, probabilistic fatigue life prediction procedure in the case of that the fatigue loading obeys lognormal distribution will be taken as an example. Suppose that $\ln(\Delta\sigma) \sim N\left(\mu_1, \sigma_1^2\right)$, that is the probability density function of fatigue loading is described as

$$f(\Delta\sigma; \mu_1, \sigma_1) = \frac{1}{(\Delta\sigma)\sigma_1\sqrt{2\pi}} \exp\left(-\frac{(\ln(\Delta\sigma) - \mu_1)^2}{2\sigma_1^2}\right) \tag{23}$$

Now, taking the natural logarithm of both sides of equation (20), it is rewritten as

$$\ln a_{eqv} = \ln k + b\ln(\Delta\sigma) \tag{24}$$

Thus, the logarithm of equivalent crack size follows normal distribution with the mean $\mu_2 = b\mu_1 + \ln k$ and variance $\sigma_2^2 = b^2\sigma_1^2$, namely $\ln(a_{eqv}) \sim N(\mu_2, \sigma_2^2)$. So the probability density function of the equivalent crack size can be expressed as

$$f(a_{eqv}; \mu_1, \sigma_1) = \frac{1}{(a_{eqv})b\sigma_1\sqrt{2\pi}}\exp\left(-\frac{(\ln(a_{eqv}) - (b\mu_1 + \ln c))^2}{2b^2\sigma_1^2}\right) \tag{25}$$

So, the probability density function of the equivalent crack size can be conveniently obtained according to loading condition. Subsequently, probabilistic total fatigue life can be easily derived.

Probabilistic total fatigue life prediction based on equivalent crack size determination

As aforementioned, when random loading is applied, probabilistic total fatigue life can be predicted once the probability distribution function of equivalent crack size is obtained. Therefore, the equivalent crack size is the key point for probabilistic total fatigue life prediction. In the last section, the assumption that the equivalent crack size relies on load amplitude is proposed, and the exponential relationship between them, which is described by equation (20) is supposed based on the research works in Molent et al. (2008). The feasibility and reliability of the proposed method for determining the equivalent crack size is discussed in this section.

According to the experimental data listed in Table 1, the equivalent crack size a_{eqv} under the four sets of loading conditions, they are $\Delta\sigma = 225$, 175, 180, and 140 MPa, can be calculated. To obtain more accurate statistical characteristics, the equivalent crack size a_{eqv} is also calculated in the case that stress ratio is taken as $R = 0.1$, and load amplitude are 500, 450, 400, 350, 300, and 250 MPa, separately. Putting load amplitude and the equivalent crack size in Figure 3, and taking the former as a vertical coordinate and the latter as a horizontal coordinate. In this way, the equivalent crack size decreases as load amplitude increases, and the relationship between them can be best fitted by an exponential function, as shown in Figure 4.

From Figure 4, the equivalent crack size decreases exponentially with the increase of load amplitude. The relationship can be fitted by

$$a_{eqv} = 3.181(\Delta\sigma)^{-0.7642} \tag{26}$$

Therefore, there is an exponential relation between equivalent crack size and load amplitude, that is the method to determine the equivalent crack size expressed by equation (26), which is proposed in this paper is reliable. Thus, the relation can be used to predict probabilistic total fatigue life. Namely, once the load condition is known, the equivalent crack size a_{eqv} can be determined by using equation (26). Then, total fatigue life is derived as

$$N_{total} = \int_{a_{eqv}}^{a_f} \frac{\mathrm{d}a}{C(\Delta K)^m} = \int_{a_{eqv}}^{a_f} \frac{\mathrm{d}a}{C(Y\Delta\sigma\sqrt{\pi a})^m} = \int_{3.181(\Delta\sigma)^{-0.7642}}^{a_f} \frac{\mathrm{d}a}{C(Y\Delta\sigma\sqrt{\pi a})^m} \tag{27}$$

In this way, the total fatigue life is described as a function of the load amplitude and the welded property. For a certain kind of welded joint (16Mn steel welded joints in this case), probabilistic total fatigue life prediction results can be obtained by combining equation (27) and probability density functions of load amplitude and equivalent crack size.

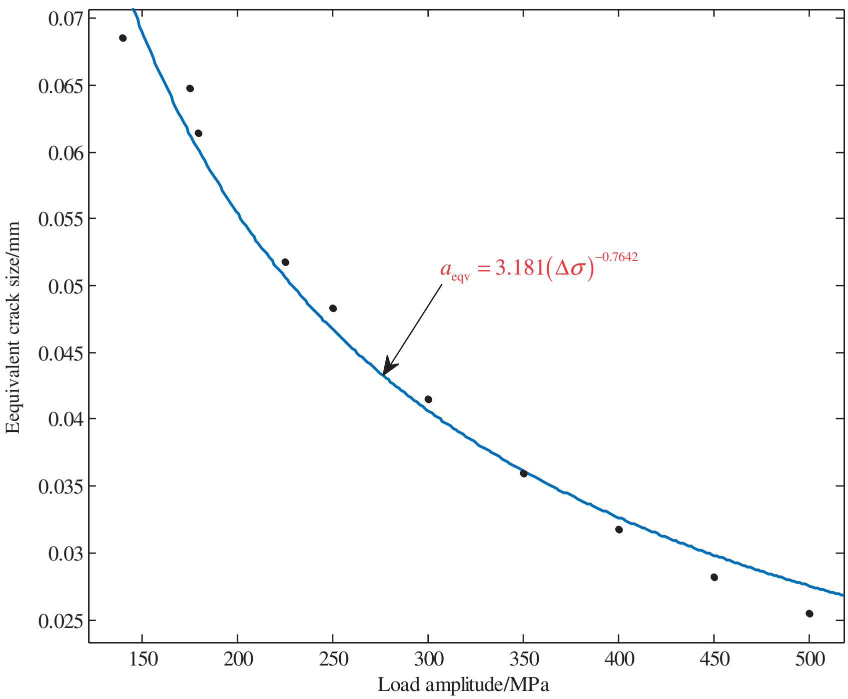

Figure 4. Equivalent crack size versus load amplitude and fitting result.

Based on the above analysis, probabilistic total fatigue life will be predicted when random loading is applied to 16Mn steel welded joints, with the relation between equivalent crack size and load amplitude. The load amplitude is often regarded as in lognormal or Weibull distribution in current study (Altamura and Straub, 2014; Prasad et al., 2013). Thus, the supposition that load amplitude follows lognormal distribution with the parameters of 5.4161 and 0.1, that is $\ln(\Delta\sigma) \sim N(5.4161,\ 0.1)$ is first assumed here. Thus, according to equation (26), the equivalent crack size also follows lognormal distribution, $\ln\left(a_{eqv}\right) \sim N(-2.9817,\ 0.05840)$. Then, probabilistic total fatigue life can be obtained by Monte Carlo sampling simulation, as shown in Figure 5. The probabilistic total fatigue life calculated by equation (18) can be also seen in Figure 5. From Figure 4, it can be found that total fatigue life presented by the two models are approximate, that is the predictions of the proposed method fluctuate around those of equation (18). For small failure probability, there is a relatively large distance, but the distance is decreased with increasing failure probability, and high failure probability corresponds to the result that the predictions of the proposed method are more conservative than those calculated by equation (18).

Figure 6 shows the comparison results calculated by these two methods, which is under the supposition that load amplitude follows Weibull distribution with shape parameter of 225 and scale parameter of 30. And the same phenomenon can also be found in the case of load amplitude with the Weibull distribution. That is, the calculation process of the proposed method is simpler, but this method can produce approximate probabilistic total fatigue life predictions. Thus, the consideration that probabilistic total fatigue life predicted by the proposed method in "Probabilistic total fatigue life prediction under random loading based on equivalent crack size determination" section is reliable and feasible. By using "normal probability plot" command in MATLAB software,

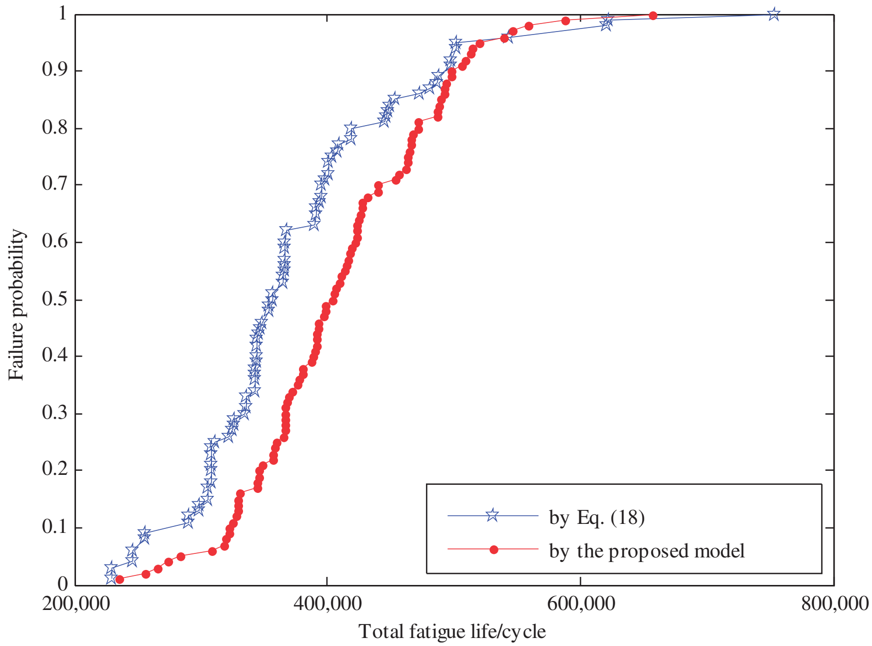

Figure 5. Comparison results of probabilistic total fatigue life calculated by equation (18) and the proposed method in "Probabilistic total fatigue life prediction under random loading based on equivalent crack size determination" section when load amplitude follows lognormal distribution.

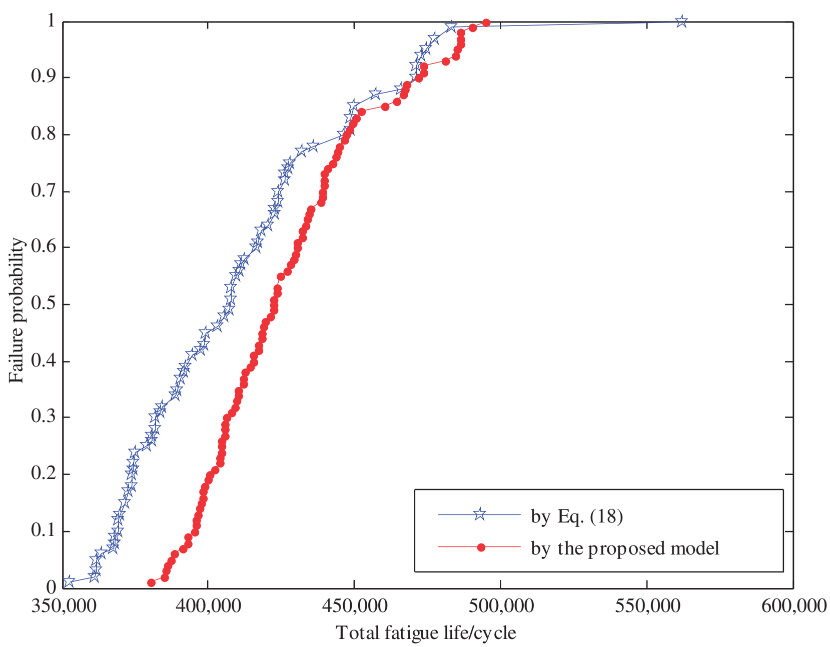

Figure 6. Comparison results of probabilistic total fatigue life calculated by equation (18) and the proposed method in "Probabilistic total fatigue life prediction under random loading based on equivalent crack size determination" section when load amplitude follows Weibull distribution.

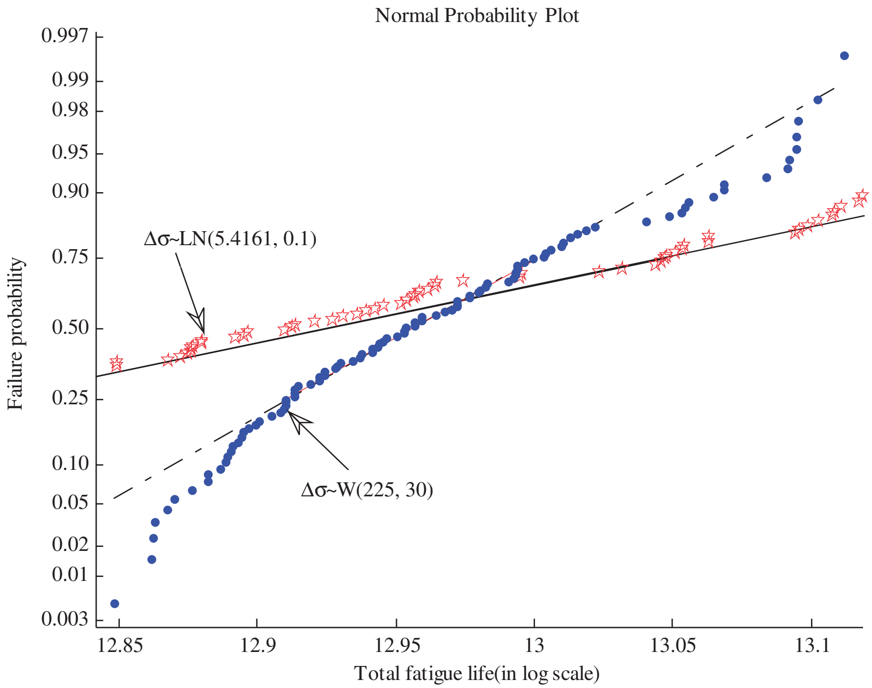

Figure 7. Normal distribution test results.

the total fatigue life predicted by the proposed method follows lognormal distribution, can be accepted, as shown in Figure 7. And through fitting analysis, $\ln(N_{total})$ can be considered as a random variable in normal distribution with the mean of 12.9078 and the variance of 0.1830 when $\ln(\Delta\sigma) \sim N(5.4161, 0.1)$, and under the assumption that $\Delta\sigma \sim W(225, 30)$, $\ln(N_{total})$ can also be considered as a random variable in normal distribution with the mean of 12.9621 and the variance of 0.06519. It should be noted that although the comparison results in Figures 5 and 6 show that the predictions of the proposed method fluctuate around those of equation (18), the proposed method greatly improves the computational efficiency and is easy to use but sacrifices a certain reliability. How to assure the reliability and improve the calculation efficiency is an issue that needs to be resolved, which is worthy of further research.

Conclusion

Deterministic and probabilistic total fatigue life prediction methods for welded joints are investigated in this paper. First, a new method is proposed to determine initial crack size, and deterministic total fatigue life can be further predicted. In the proposed method, initial crack size is related to loading conditions, which can reflect the research results of the back-extrapolation method. Because the fictitious values are not required in the whole process, human factors and uncertainties can be reduced. And the time and cost can also be reduced because fewer tests are needed. Compared with the common methods for deterministic total fatigue life prediction, the proposed method can give a definitive result, and there is no confusion for initial crack size determination. In addition, the results predicted by the proposed procedure yield good agreement with experimental fatigue life, and the applicability of the proposed method is verified.

Based on the initial crack size-based procedure, equivalent crack size is presented to predict probabilistic total fatigue life conveniently under random loading, which defines the equivalent crack size as an exponential function of load amplitude. Numerical simulation results show that the total fatigue life predictions for 16Mn steel welded joints can be described by lognormal distribution, when load amplitude with lognormal or Weibull distribution is applied. When determining the equivalent crack size, the proposed method only needs a few sets of experimental data and can avoid the calculation process of initial crack size. According to the comparison analysis, the accuracy of the probabilistic total fatigue life predictions based on the proposed method to determine the equivalent crack size is acceptable.

Future work on determination of the material parameters in the expression of equivalent crack size is needed for more accurate and simpler predicted deterministic and probabilistic total fatigue life of welded joints.

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Appendix

Notation

a	crack depth
a_{eqv}	equivalent crack size
a_{p0}	initial crack size

C, m	material parameters
E	Young's modulus
k, b	material constants
K	strength coefficient
K_t	stress concentration factor
n	strain hardening exponent
N_f	fatigue life
N_i	crack initiation life
N_p	fatigue propagation life
N_{total}	total fatigue life
R	stress ratio
$2c$	crack length
Y	geometry correction factor
ΔK	stress intensity factor
ΔK_{th}	fatigue crack growth threshold
$\Delta\varepsilon$	cyclic strain range
$\Delta\varepsilon_c$	theoretical strain fatigue limit
$\Delta\sigma$	load amplitude
$\Delta\sigma_{eqv}$	equivalent load amplitude
$\left(\Delta\sigma_{eqv}\right)_{th}$	fatigue crack initial threshold
ε_f	fracture ductility
σ_f	fracture strength
σ_{-1}	fatigue limit